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Zero On-Axis Backscattering of an Anisotropically Coated Shell of Revolution

by

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A sufficient condition for eliminating the on-axis backscattering of an anisotropic impedance coated shell of revolution has been deduced. The outside and inside normalized surface impedances Z^+ and Z^- with which this sufficient condition can be satisfied have all been found. One exceptional situation is when the impedance matrices are equal and skew-symmetric with their determinants equal to -1. All other cases require that the two matrices be symmetric, their determinants be unity, and the determinant of their difference be zero. The shell under consideration can be a closed one. For such a body of revolution only the conditions on Z^+ need to apply, i.e., Z^+ must be either symmetric or skew-symmetric, with $\det[Z^+] = \pm 1$. This is an extension of Weston's result to anisotropically coated bodies. Results of this work make available a wide class of models which must have zero on-axis backscattering cross section. All general purpose numerical codes for computing the scattering cross sections of anisotropic impedance coated objects should be checked for their accuracy against a selected group of such models. Such comparisons should provide indications of an error bound of the particular algorithm.

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I. INTRODUCTION

The electromagnetic scattering from an arbitrarily shaped shell of revolution has been investigated. An example of the geometry is shown in Fig. 1. The surface is parameterized in the cylindrical coordinate by $(\rho_g(s),\ \phi,\ z_g(s))$ with $0 \le s \le l$ and $0 \le \phi \le 2\pi$, where the z-axis is the axis of rotation and s is the arc length parameter of the generating curve (Fig. 2). Henceforth the subscript "g" will denote the described on the generating curve. Also shown in Fig. 2 are the unit tangent $\hat{t}$ and the unit normal $\hat{n}^+ = \hat{\phi} \times \hat{t}$ to the curve. The parameterization of the generating curve is chosen so that $\hat{n}^+$ is the outward normal of the outer surface of the shell. The angle θ_g, measured from the positive $\hat{z}$ axis to $\hat{t}$, unlike the polar angle in a spherical coordinate system, can assume negative values or positive values greater than π.

The inner and outer surfaces of the perfectly conducting shell are coated with anisotropic materials which can be different. It is assumed that the thickness of the coating and of the shell are infinitesimal. The coated surfaces are also assumed to satisfy the impedance boundary condition (IBC). It is found that, with some special impedance matrices, the backscattering cross section along the axis of this coated shell can be eliminated. This finding will be presented in this report.

For convenience in both theoretical formulation and numerical computation, it is desirable to consider $\vec{E}$ as the electric field intensity vector divided by the intrinsic impedance $\eta = \sqrt{\mu/\varepsilon}$ of the isotropic, homogeneous medium within which the shell is located. Therefore, electric field intensity $\vec{E}$ takes the unit of amperes per meter, the same as those of the magnetic field

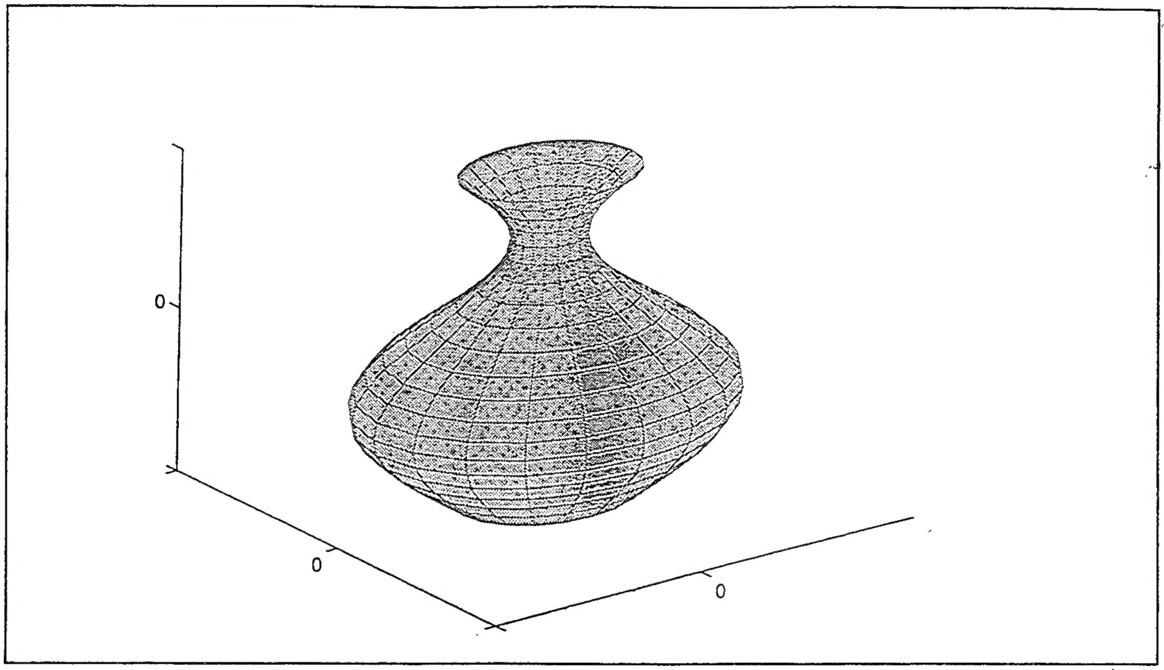

Figure 1. An arbitrarily shaped shell of revolution.

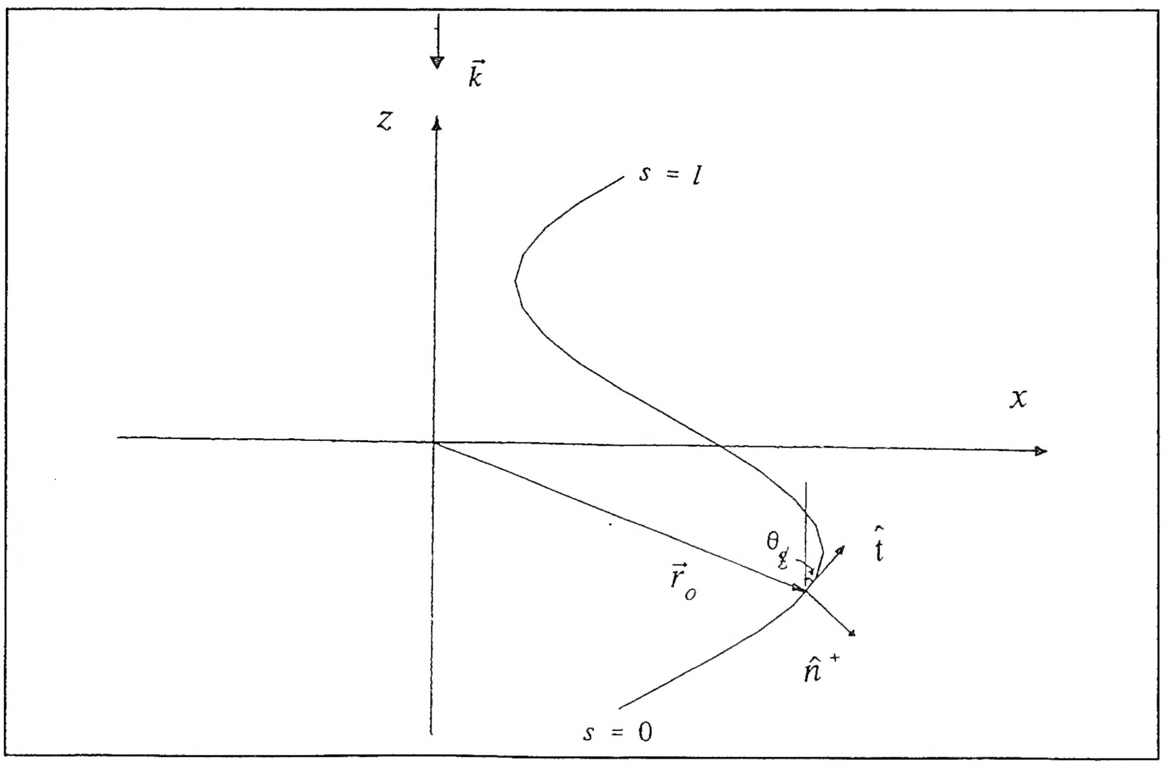

Figure 2. The geometry and coordinates of the generating curve.

intensity $\vec{H}$ and the electric and the magnetic surface current distributions. The surface impedances, Z^+ on the outside and Z^- on the inside of the shell, are also normalized with the same factor to become dimensionless. The time dependence $e^{-i\omega t}$ will be assumed and suppressed. These are the conventions adopted in this report.

For the remainder of this chapter, the major steps taken to prove the vanishing of on-axis backscattering cross section are briefly sketched in Sections A through D. The sufficient condition to eliminate this cross section is specified at the end of Sec. D. In Sec. E, the impedances with which the on-axis backscattering can be eliminate are given. They are the main contributions of this work. All general purpose programs [1, 2] for the numerical computation of scattering cross sections of anisotropic impedance coated objects should be checked for their accuracy against selected surfaces of revolution coated with impedance matrices satisfying the conditions listed in that section. The results will provide an indication of the error bound for the particular algorithm.

A. SCATTERED FIELD

The Stratton- Chu equations [3] give the radiation in a homogeneous and isotropic region outside a closed surface S in terms of the tangential components of the $\vec{E}$ and $\vec{H}$ field intensities on S. The scattered fields $\vec{E}^{sc}$ and $\vec{H}^{sc}$ at a point $\vec{r}$ outside S which encloses the shell are given by:

$$\begin{aligned}4\pi\vec{E}^{sc}(\vec{r}) \;=\; & k\nabla \times \int_S [\hat{n}\times\vec{E}(\vec{r}_o)]G(\vec{r}-\vec{r}_o)da_o + ik^2 \int_S [\hat{n}\times\vec{H}(\vec{r}_o)]G(\vec{r}-\vec{r}_o)da_o \\ & - i\nabla \int_S [\hat{n}\times\vec{H}(\vec{r}_o)]\bullet\nabla_o G(\vec{r}-\vec{r}_o)da_o\end{aligned} \tag{1}$$

$$4\pi\vec{H}^{sc}(\vec{r}) = k\nabla \times \int_S [\hat{n}\times\vec{H}(\vec{r}_o)]G(\vec{r}-\vec{r}_o)da_o - ik^2 \int_S [\hat{n}\times\vec{E}(\vec{r}_o)]G(\vec{r}-\vec{r}_o)da_o$$
$$+ i\nabla \int_S [\hat{n}\times\vec{E}(\vec{r}_o)]\circ\nabla_o G(\vec{r}-\vec{r}_o)da_o \qquad \textbf{(2)}$$

where $\hat{n}$ is the unit outward normal of S, $k = \omega\sqrt{\mu\varepsilon}$, and $G(\vec{r}-\vec{r}_o)=\dfrac{e^{ik|\vec{r}-\vec{r}_o|}}{k|\vec{r}-\vec{r}_o|}$. Note that the tangential components of $\vec{E}$ and $\vec{H}$ are the equivalent magnetic and electric surface current distributions on S (rotated by 90^o.)

In eqs. (1) and (2), the surface S can be shrunk toward the shell. We denote the outer surface of the shell as S^+, and the inner surface of the shell as S^-. Henceforth the superscripts "+" and "-" will denote the described values on S^+ and S^- respectively. The total (incident and scattered) equivalent electric currents $\vec{K}^+$ on S^+ and $\vec{K}^-$ on S^- are:

$$\vec{K}^+ = \hat{n}^+ \times \vec{H}^+$$
$$\vec{K}^- = \hat{n}^- \times \vec{H}^- = -\hat{n}^+ \times \vec{H}^- \qquad \textbf{(3)}$$

where $\hat{n}^+$ and $\hat{n}^-$ are the unit outward normals on S^+ and S^- respectively. $\vec{H}^+$ and $\vec{H}^-$ are the total magnetic field intensities. Because the thickness of the shell is considered infinitesimal, $\hat{n}^+ = -\hat{n}^-$ at any point of the shell.

Similarly, the equivalent magnetic currents $\vec{L}^+$ and $\vec{L}^-$ are:

$$\vec{L}^+ = \vec{E}^+ \times \hat{n}^+$$
$$\vec{L}^- = \vec{E}^- \times \hat{n}^- = \vec{E}^- \times (-\hat{n}^-) \qquad \textbf{(4)}$$

Define $\vec{K}(\phi_o, s_o)$ as the sum of the outside and inside total electric surface currents, and similarly $\vec{L}(\phi_o, s_o)$ as the sum of the outside and inside total magnetic surface currents:

$$\vec{K} = \vec{K}^{+} + \vec{K}^{-} \tag{5}$$

$$\vec{L} = \vec{L}^{+} + \vec{L}^{-} \tag{6}$$

Since the thickness of the coating and of the shell are assumed to be infinitesimal, the contribution from S_o and S_l (Fig. 3) to the integrals must vanish by the edge condition [5] as the distance of S to the shell becomes zero. Eqs. (1) and (2) are reduced to:

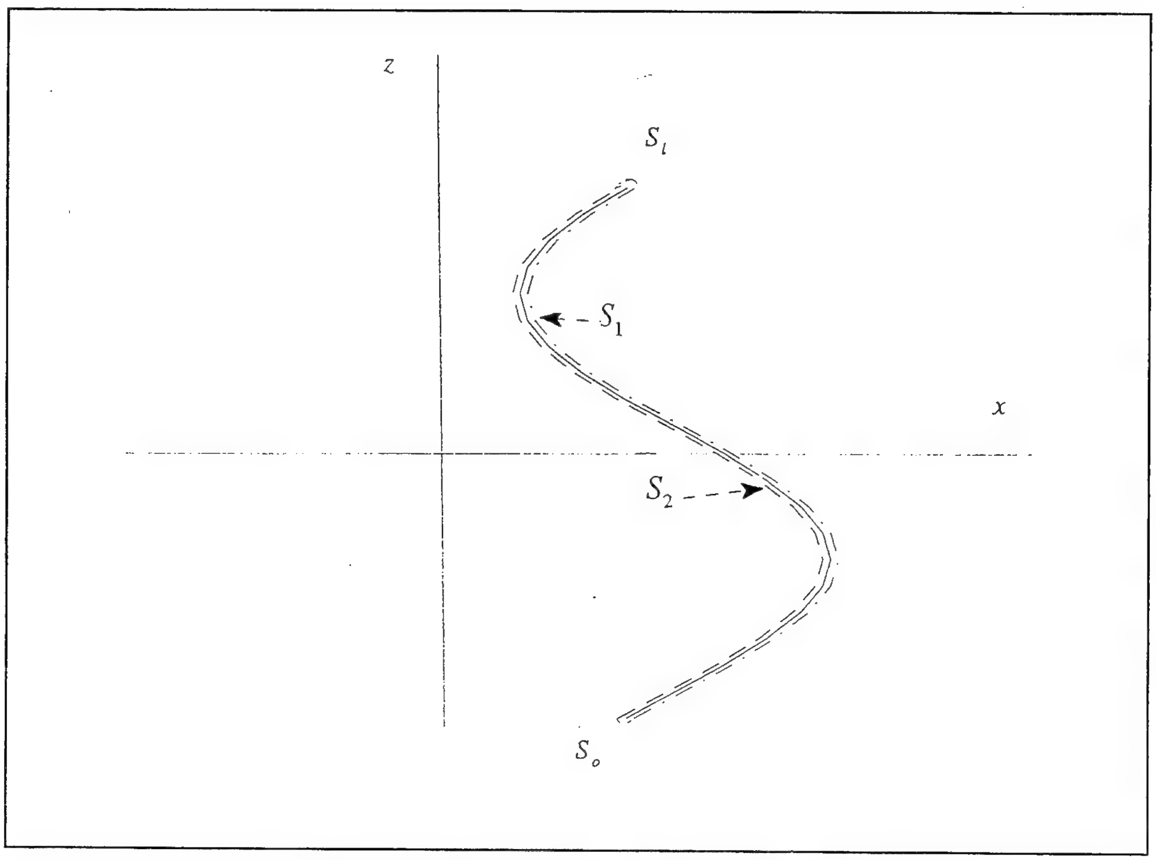

Figure 3. Shrink S to the surface of the shell.

$$
\begin{aligned}
4\pi\vec{E}^{sc}(\vec{r}) = & -k\nabla\times\int_0^l\int_0^{2\pi}\vec{L}(\phi_o,s_o)G(\vec{r}-\vec{r}_o)\rho_o ds_o d\phi_o \\
& + ik^2\int_0^l\int_0^{2\pi}\vec{K}(\phi_o,s_o)G(\vec{r}-\vec{r}_o)\rho_o ds_o d\phi_o \\
& - i\nabla\int_0^l\int_0^{2\pi}\vec{K}(\phi_o,s_o)\bullet\nabla_o G(\vec{r}-\vec{r}_o)\rho_o ds_o d\phi_o
\end{aligned}
\tag{7}
$$

and

$$
\begin{aligned}
4\pi\vec{H}^{sc}(\vec{r}) = & \; k\nabla\times\int_0^l\int_0^{2\pi}\vec{K}(\phi_o,s_o)\,G(\vec{r}-\vec{r}_o)\rho_o ds_o d\phi_o \\
& + ik^2\int_0^l\int_0^{2\pi}\vec{L}(\phi_o,s_o)\,G(\vec{r}-\vec{r}_o)\rho_o ds_o d\phi_o \\
& - i\nabla\int_0^l\int_0^{2\pi}\vec{L}(\phi_o,s_o)\bullet\nabla_o G(\vec{r}-\vec{r}_o)\rho_o ds_o d\phi_o
\end{aligned}
\tag{8}
$$

Note that the scattered fields are determined completely by the sum currents on the shell.

In the far-field region, the scattered field $\vec{E}^{sc}(\vec{r})$ can be expressed in the spherical coordinate system as:

$$
\begin{aligned}
\vec{E}^{sc}(\vec{r}) \approx ik\frac{e^{ikr}}{4\pi r}\int_0^l ds_o\int_0^{2\pi}d\phi_o\ \rho_o[\ & \hat{\theta}[K_t\sin\theta_g\cos\theta\cos(\phi-\phi_o)+K_\phi\cos\theta\sin(\phi-\phi_o) \\
& -K_t\cos\theta_g\sin\theta+L_\phi\cos(\phi-\phi_o)-L_t\sin\theta_g\sin(\phi-\phi_o)] \\
& +\hat{\phi}[K_\phi\cos(\phi-\phi_o)-K_t\sin\theta_g\sin(\phi-\phi_o) \\
& -L_t\sin\theta_g\cos\theta\cos(\phi-\phi_o)+L_t\cos\theta_g\sin\theta \\
& -L_\phi\cos\theta\sin(\phi-\phi_o)]\ ]e^{-i[kz_o\cos\theta+k\rho_o\sin\theta\cos(\phi-\phi_o)]}
\end{aligned}
\tag{9}
$$

where the subscripts ϕ and t denote the tangential components of the surface currents in the $\hat{\phi}$ and $\hat{t}$ directions on the surface of the shell respectively. Because of the rotational symmetry of the shell, the Fourier expansion can be utilized to solve the ϕ-dependence of this problem. Define the Fourier series component $f_n\ (s)$ of a function $f\ (\phi,s)$ by:

$$f(\phi,s) = \sum_{n=-\infty}^{\infty} e^{in\phi} f_n(s) \tag{10}$$

then as $\theta \to 0$, eq. (9) is simplified and transformed to:

$$\begin{aligned}\vec{E}^{sc}(\vec{r}) = k\,\frac{e^{ikr}}{4r}\int_0^l ds_o\ \rho_o e^{-ikz_o}\{&[(K_{\phi,1}+iL_{\phi,1})+i(K_{t,1}+iL_{t,1})\sin\theta_g](\hat{x}+i\hat{y})\\ &-[(K_{\phi,-1}-iL_{\phi,-1})-i(K_{t,-1}-iL_{t,-1})\sin\theta_g](\hat{x}-i\hat{y})\}\end{aligned} \tag{11}$$

The backscattering cross section is given by:

$$\sigma_B = \lim_{r\to\infty} 4\pi r^2 \frac{|\vec{E}^{sc}|^2}{|\vec{E}^{inc}|^2} \tag{12}$$

For an incident plane wave directed along the axis of the shell, with $|\vec{E}^{inc}|$ normalized to unity, σ_B becomes:

$$\begin{aligned}\sigma_B = \frac{\pi k^2}{4}\ \Big|\int_0^l ds_o\ \rho_o e^{-ikz_o}\{&[(K_{\phi,1}+iL_{\phi,1})+i(K_{t,1}+iL_{t,1})\sin\theta_g](\hat{x}+i\hat{y})\\ &-[(K_{\phi,-1}-iL_{\phi,-1})-i(K_{t,-1}-iL_{t,-1})\sin\theta_g](\hat{x}-i\hat{y})\}\Big|^2\end{aligned} \tag{13}$$

B. IMPEDANCE BOUNDARY CONDITION

The impedance boundary condition links the tangential components of the total electric field to those of the total magnetic field on a surface through its surface impedance which is a function of the electromagnetic properties of the material, i.e.,

$$\hat{n}^{\pm} \times (\vec{E}^{\pm} \times \vec{n}^{\pm}) = Z^{\pm} (\hat{n}^{\pm} \times \vec{H}^{\pm}) \tag{14}$$

where Z^+ and Z^- are the normalized surface on S^+ and S^- respectively. Equivalently,

$$\hat{n}^{\pm} \times \vec{L}^{\pm} = Z^{\pm} \vec{K}^{\pm} \tag{15}$$

For anisotropic materials, it is more convenient to utilize matrix notations. In eq. (15), the vectors $\vec{K}^{\pm}$ and $\vec{L}^{\pm}$ can be considered as two-element column vectors, with each of the ϕ - component designated as element 1 and the t - component as element 2. Z^+ and Z^- are then two-by-two matrices. The cross product of the unit vector $\hat{n}^+$ with such a two-element column vector can be represented with the following matrix multiplication:

$$\hat{n}^{+} \times \vec{L}^{\pm} = V \vec{L}^{\pm} \tag{16}$$

where $V = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Note that $V^T = -V$ and $V^2 = -I$.

In terms of the matrix V, the impedance boundary condition, eq. (15), can be written as:

$$\begin{aligned} Z^+\vec{K}^+ &= V\vec{L}^+ \\ Z^-\vec{K}^- &= -V\vec{L}^- \end{aligned} \tag{17}$$

Since the scattered field is determined completely by the sum currents, it is desirable to write the impedance boundary condition as one which provides the difference currents in terms of the sum currents. Define:

$$Z = \frac{1}{2}\,(Z^{+} + Z^{-}) \tag{18}$$

and

$$\Delta = \frac{1}{2}\,(Z^{+} - Z^{-}) \tag{19}$$

then, assuming that Z is invertible, eq. (17) can be transformed into the following two equations:

$$\vec{K}^{+} - \vec{K}^{-} = Z^{-1}V\,\vec{L} - Z^{-1}\Delta\,\vec{K} \tag{20}$$

$$\vec{L}^{+} - \vec{L}^{-} = -V\,[Z - \Delta Z^{-1}\Delta]\,\vec{K} - V\Delta Z^{-1}V\vec{L} \tag{21}$$

C. EQUATIONS FOR SCATTERING CURRENTS

From eq. (3), using matrix notation and noting that $\hat{n} \times \vec{H} = \hat{n} \times \vec{H}_{\tan}$ on the surface of the shell, one can deduce that:

$$\begin{aligned} \vec{K}^{+} - \vec{K}^{-} &= V\,\vec{H}^{+}_{\tan} - (-V)\,\vec{H}^{-}_{\tan} \\ &= V(\vec{H}^{inc}_{\tan} + \vec{H}^{sc+}_{\tan}) + V(\vec{H}^{inc}_{\tan} + \vec{H}^{sc-}_{\tan}) = V\,(\vec{H}^{sc+}_{\tan} + \vec{H}^{sc-}_{\tan} + 2\vec{H}^{inc}_{\tan}) \end{aligned} \tag{22}$$

therefore,

$$\vec{H}^{sc+}_{\tan} + \vec{H}^{sc-}_{\tan} + 2\vec{H}^{inc}_{\tan} = -V\,(\vec{K}^{+} - \vec{K}^{-}) \tag{23}$$

Similarly,

$$\vec{E}^{sc+}_{\tan} + \vec{E}^{sc-}_{\tan} + 2\vec{E}^{inc}_{\tan} = V\,(\vec{L}^{+} - \vec{L}^{-}) \qquad \textbf{(24)}$$

The Fourier coefficients of the tangential components of the scattered fields approaching the outer and inner surfaces of the shell in eqs. (23) and (24) can be obtained from eqs. (7) and (8) and written as:

$$\vec{E}^{sc+}_{\tan,n}(s) + \vec{E}^{sc-}_{\tan,n}(s) = -M_n\,\vec{K}_n(s) + N_n\,\vec{L}_n(s) \qquad \textbf{(25)}$$

$$\vec{H}^{sc+}_{\tan,n}(s) + \vec{H}^{sc-}_{\tan,n}(s) = -N_n\,\vec{K}_n(s) - M_n\,\vec{L}_n(s) \qquad \textbf{(26)}$$

The elements of the two-by-two matrices M_n and N_n are integrodifferential operators on the elements of the column vectors $\vec{K}_n$ and $\vec{L}_n$. They are derived and given later in eqs. (85) and (86).

Combining these with eqs (20) and (21):

$$\left\{ \begin{bmatrix} M_n & -N_n \\ N_n & M_n \end{bmatrix} + \begin{bmatrix} Z-\Delta Z^{-1}\Delta & \Delta Z^{-1}V \\ VZ^{-1}\Delta & -VZ^{-1}V \end{bmatrix} \right\} \begin{bmatrix} \vec{K}_n \\ \vec{L}_n \end{bmatrix} = 2 \begin{bmatrix} \vec{E}^{inc}_{\tan,n} \\ \vec{H}^{inc}_{\tan,n} \end{bmatrix} \qquad \textbf{(27)}$$

where unlike the products of M_n, N_n with $\vec{K}_n$, $\vec{L}_n$, the matrices involving the impedances are constants and the product with $\vec{K}_n$, $\vec{L}_n$ are usual matrix multiplications. Given the incident wave, this is a linear integral equation of the second kind for the unknown currents $\vec{K}_n$, $\vec{L}_n$. It is assumed that unique solutions to eq. (27) exist for all n.

D. ZERO ON-AXIS BACKSCATTERING CONDITION

Because of the rotational symmetry, a wave incident along the axis of the shell (the z-axis) can be assumed to be linearly polarized in the y-direction without loss of generality. Let the fields on the surface of the shell be:

$$
\begin{aligned}
\vec{E}^{\,inc}(\phi,s) &= \hat{y}\, e^{-ikz_g} = [\,\cos\phi\hat{\phi} + \sin\phi\,(\sin\theta_g \hat{t} + \cos\theta_g \hat{n}^{+})\,]\, e^{-ikz_g(s)} \\
\vec{H}^{\,inc}(\phi,s) &= \hat{x}\, e^{-ikz_g} = [-\sin\phi\hat{\phi} + \cos\phi\,(\sin\theta_g \hat{t} + \cos\theta_g \hat{n}^{+})\,]\, e^{-ikz_g(s)}
\end{aligned}
\qquad \textbf{(28)}
$$

where the superscript "*inc*" denotes the incident wave.

The Fourier coefficients of the tangential components of the incident fields are:

$$
\begin{cases}
E_{\phi,n}^{inc} = [\delta_{n-1} + \delta_{n+1}]\,\dfrac{e^{-ikz_g}}{2} \\
E_{t,n}^{inc} = [\delta_{n-1} - \delta_{n+1}]\,\dfrac{e^{-ikz_g}}{2i}\sin\theta_g \\
H_{\phi,n}^{inc} = [\delta_{n-1} - \delta_{n+1}]\,\dfrac{ie^{-ikz_g}}{2} \\
H_{t,n}^{inc} = [\delta_{n-1} + \delta_{n+1}]\,\dfrac{e^{-ikz_g}}{2}\sin\theta_g
\end{cases}
\qquad \textbf{(29)}
$$

Note that only the $n = \pm 1$ terms in eq. (29) are nonzero. Therefore, only the cases $n = \pm 1$ have to be considered.

Since

$$
\vec{E}_{\tan,\pm1}^{inc} = \frac{1}{2}\begin{bmatrix} 1 \\ \mp i\sin\theta_g \end{bmatrix} e^{-ikz_g}, \qquad \vec{H}_{\tan,\pm1}^{inc} = \frac{1}{2}\begin{bmatrix} \pm i \\ \sin\theta_g \end{bmatrix} e^{-ikz_g} \qquad \textbf{(30)}
$$

The incident fields satisfy the relation:

$$\left\{ 1 \mp \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \right\} \begin{bmatrix} \vec{E}^{\,inc}_{\tan,\pm 1} \\ \vec{H}^{\,inc}_{\tan,\pm 1} \end{bmatrix} = 0 \tag{31}$$

On the other hand,

$$\begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \begin{bmatrix} M_n & -N_n \\ N_n & M_n \end{bmatrix} = \begin{bmatrix} M_n & -N_n \\ N_n & M_n \end{bmatrix} \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \tag{32}$$

Therefore, if $\begin{bmatrix} Z-\Delta Z^{-1}\Delta & \Delta Z^{-1}V \\ VZ^{-1}\Delta & -VZ^{-1}V \end{bmatrix}$ also commutes with $\begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix}$, then

$$\begin{aligned}
&\left\{ \begin{bmatrix} M_{\pm 1} & -N_{\pm 1} \\ N_{\pm 1} & M_{\pm 1} \end{bmatrix} + \begin{bmatrix} Z-\Delta Z^{-1}\Delta & \Delta Z^{-1}V \\ VZ^{-1}\Delta & -VZ^{-1}V \end{bmatrix} \right\} \left\{ 1 \mp \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \right\} \begin{bmatrix} \vec{K}_{\pm 1} \\ \vec{L}_{\pm 1} \end{bmatrix} \\
&= \left\{ 1 \mp \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \right\} \left\{ \begin{bmatrix} M_{\pm 1} & -N_{\pm 1} \\ N_{\pm 1} & M_{\pm 1} \end{bmatrix} + \begin{bmatrix} Z-\Delta Z^{-1}\Delta & \Delta Z^{-1}V \\ VZ^{-1}\Delta & -VZ^{-1}V \end{bmatrix} \right\} \begin{bmatrix} \vec{K}_{\pm 1} \\ \vec{L}_{\pm 1} \end{bmatrix} \\
&= 2\left\{ 1 \mp \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \right\} \begin{bmatrix} \vec{E}^{\,inc}_{\tan,\pm 1} \\ \vec{H}^{\,inc}_{\tan,\pm 1} \end{bmatrix} \\
&= 0
\end{aligned} \tag{33}$$

The existence and uniqueness of the solution to eq. (27) implies that

$$\left\{ 1 \mp \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \right\} \begin{bmatrix} \vec{K}_{\pm 1} \\ \vec{L}_{\pm 1} \end{bmatrix} = \mathbf{0} \tag{34}$$

i.e., $\vec{K}_{\pm 1} \pm i\vec{L}_{\pm 1} = 0$. From eq. (13), $\sigma_B = 0$. There will be no backscattering.

E. IMPEDANCE MATRICES

It has been deduced that a sufficient condition to eliminate on-axis backscattering is:

$$\begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \begin{bmatrix} Z-\Delta Z^{-1}\Delta & \Delta Z^{-1}V \\ VZ^{-1}\Delta & -VZ^{-1}V \end{bmatrix} = \begin{bmatrix} Z-\Delta Z^{-1}\Delta & \Delta Z^{-1}V \\ VZ^{-1}\Delta & -VZ^{-1}V \end{bmatrix} \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \tag{35}$$

which is equivalent to:

$$VZ^{-1}\Delta = -\Delta Z^{-1}V \tag{36}$$

and

$$Z + VZ^{-1}V = \Delta Z^{-1}\Delta \tag{37}$$

To satisfy these two conditions, either

$$Z^{+} = Z^{-} = \begin{bmatrix} 0 & \pm i \\ \mp i & 0 \end{bmatrix} \tag{38}$$

when Z^{+} and Z^{-} are skew-symmetric, or, in the case when Z^{+} and Z^{-} are symmetric,

$$\det Z^{+} = \det Z^{-} = 1 \tag{39}$$

$$\det(Z^{+} - Z^{-}) = 0 \tag{40}$$

Note that when the shell is a closed surface, the impedance boundary condition separates the interior volume from the solution in the exterior. Only the conditions on Z^+ will apply to the exterior problem. Therefore, for an anisotropic impedance coated body of revolution, the on-axis backscattering cross section vanishes either $Z^+ = \begin{bmatrix} 0 & \pm i \\ \mp i & 0 \end{bmatrix}$ or $\det Z^+ = 1$ with a symmetric Z^+.

II. SCATTERING OF A COATED SHELL OF REVOLUTION

A. RADIATION FROM EQUIVALENT CURRENTS

The geometry and coordinate system for an arbitrarily shaped shell of revolution are shown in Figures 1 and 2. In the cylindrical coordinates (ρ, ϕ, z), the shell is described by $\rho = \rho_g(s)$, $z = z_g(s)$ with $0 \le s \le l$ where s is the arc length parameter of the generating curve. Since both ρ and z are functions of s, the surface of the shell is parameterized in the coordinates ϕ and s. The transformations of vectors between the cylindrical coordinates and the surface coordinates (ϕ, s) are given in Appendix A.

The Stratton-Chu equations give the radiation in a homogeneous and isotropic region outside a closed surface in terms of the $\vec{E}$ and $\vec{H}$ field intensities on that surface. Using the conventions adopted in this report, the radiated fields $\vec{E}^{sc}$ and $\vec{H}^{sc}$ at a point $\vec{r}$ outside S which encloses the shell of revolution can be written as [4]:

$$4\pi\vec{E}^{sc}(\vec{r}) = ik^2 \int_S (\hat{n}\times\vec{H}(\vec{r}_o))G(\vec{r}-\vec{r}_o)da_o + k \int_S (\hat{n}\times\vec{E}(\vec{r}_o)) \times \nabla_o G(\vec{r}-\vec{r}_o)da_o$$
$$+ k \int_S (\hat{n} \bullet \vec{E}(\vec{r}_o)) \nabla_o G(\vec{r}-\vec{r}_o)da_o \quad \textbf{(41)}$$

$$4\pi\vec{H}^{sc}(\vec{r}) = - ik^2 \int_S (\hat{n}\times\vec{E}(\vec{r}_o))G(\vec{r}-\vec{r}_o)da_o + k \int_S (\hat{n}\times\vec{H}(\vec{r}_o)) \times \nabla_o G(\vec{r}-\vec{r}_o)da_o$$
$$+ k \int_S (\hat{n} \bullet \vec{H}(\vec{r}_o)) \nabla_o G(\vec{r}-\vec{r}_o)da_o \quad \textbf{(42)}$$

where $k = \omega\sqrt{\mu\epsilon}$ and $G(\vec{r}-\vec{r}_o) = \dfrac{e^{ik|\vec{r}-\vec{r}_o|}}{k|\vec{r}-\vec{r}_o|}$, and $\hat{n}$ is the outward normal of S.

Eqs. (41) and (42) can be converted into ones involving integrals of tangential components of the fields only. Consider the last term in eq. (41):

$$
\begin{aligned}
k\int_S(\hat{n}\bullet\vec{E})\,\nabla_o G\,da_o &= -k\int_S(\hat{n}\bullet\vec{E})\,\nabla G\,da_o\\
&= -k\,\nabla\int_S(\hat{n}\bullet\vec{E})G\,da_o\\
&= -i\,\nabla\int_S\hat{n}\bullet(\nabla_o\times\vec{H})G\,da_o\\
&= -i\,\nabla\int_S\hat{n}\bullet[\nabla_o\times(\vec{H}\,G)-(\nabla_o G\times\vec{H})]\,da_o\\
&= -i\,\nabla\int_S\nabla_o\bullet[\nabla_o\times(\vec{H}\,G)]dv_o\,-\,i\nabla\int_S(\hat{n}\times\vec{H})\bullet\nabla_o G\,da_o\\
&= -i\,\nabla\int_S(\hat{n}\times\vec{H})\bullet\nabla_o G\,da_o
\end{aligned}
\qquad \textbf{(43)}
$$

Note that $\nabla G(\vec{r}-\vec{r}_o) = -\nabla_o G(\vec{r}-\vec{r}_o)$. The second term in eq. (41) can be written as:

$$
\int_S(\hat{n}\times\vec{E}(\vec{r}_o))\times\nabla_o G(\vec{r}-\vec{r}_o)\,da_o = \nabla\times\int_S(\hat{n}\times\vec{E}(\vec{r}_o))\,G(\vec{r}-\vec{r}_o)\,da_o \qquad \textbf{(44)}
$$

Consequently, eq. (41) is converted to:

$$
\begin{aligned}
4\pi\vec{E}^{\,sc}(\vec{r}) = {} & k\,\nabla\times\int_S[\hat{n}\times\vec{E}(\vec{r}_o)]G(\vec{r}-\vec{r}_o)da_o + ik^2\int_S[\hat{n}\times\vec{H}(\vec{r}_o)]G(\vec{r}-\vec{r}_o)da_o\\
& -i\,\nabla\int_S[\hat{n}\times\vec{H}(\vec{r}_o)]\bullet\nabla_o G(\vec{r}-\vec{r}_o)da_o
\end{aligned}
\qquad \textbf{(45)}
$$

From the duality principle [6], eq. (42) can be obtained from eq. (41) by replacing $\vec{E}$ with $\vec{H}$ and $\vec{H}$ with $-\vec{E}$. The corresponding expression to eq. (45) for the radiated magnetic field can be obtained similarly:

$$
\begin{aligned}
4\pi\vec{H}^{\,sc}(\vec{r}) = {} & k\,\nabla\times\int_S[\hat{n}\times\vec{H}(\vec{r}_o)]G(\vec{r}-\vec{r}_o)da_o - ik^2\int_S[\hat{n}\times\vec{E}(\vec{r}_o)]G(\vec{r}-\vec{r}_o)da_o\\
& +i\,\nabla\int_S[\hat{n}\times\vec{E}(\vec{r}_o)]\bullet\nabla_o G(\vec{r}-\vec{r}_o)da_o
\end{aligned}
\qquad \textbf{(46)}
$$

In eqs. (45) and (46), the surface S consisting of the four non-overlapping surfaces S_o, S_l, S_1, and S_2 can be shrunk to the surface of the shell so that $S_1 \rightarrow S^+$ and $S_2 \rightarrow S^-$ (Fig. 3). Since the thickness of the coating and of the shell are assumed to be infinitesimal, the edge conditions [5] guarantee that the contributions to the integrals from the bottom surface S_o and the top surface S_l must vanish in this limit (Appendix B). Therefore, the integrals in eqs. (45) and (46) need to be carried out over S^+ and S^- only.

The total (incident and scattered) equivalent electric currents $\vec{K}^+$ on S^+ and $\vec{K}^-$ on S^- are defined to be:

$$\vec{K}^+ = \hat{n}^+ \times \vec{H}^+$$
$$\vec{K}^- = \hat{n}^- \times \vec{H}^- = -\hat{n}^+ \times \vec{H}^- \tag{47}$$

and the equivalent magnetic currents $\vec{L}^+$ on S^+ and $\vec{L}^-$ on S^- are defined as:

$$\vec{L}^+ = \vec{E}^+ \times \hat{n}^+$$
$$\vec{L}^- = \vec{E}^- \times \hat{n}^- = \vec{E}^- \times (-\hat{n}^+) \tag{48}$$

With each integral in eqs. (45) and (46) written as a sum of integrals over the two surface S^+ and S^-, we can substitute the surface currents of eqs. (47) and (48) for the fields. Furthermore, define the sum electric current as:

$$\vec{K} = \vec{K}^+ + \vec{K}^- \tag{49}$$

and the sum magnetic current as:

$$\vec{L} = \vec{L}^{+} + \vec{L}^{-} \tag{50}$$

eqs. (45) and (46) are reduced to:

$$\begin{aligned} 4\pi\vec{E}^{sc}(\vec{r}) = &- k\nabla \times \int_0^l\int_0^{2\pi} \vec{L}(\phi_o,s_o)\ G(\vec{r}-\vec{r}_o)\ \rho_o ds_o d\phi_o \\ &+ ik^2 \int_0^l\int_0^{2\pi} \vec{K}(\phi_o,s_o)\ G(\vec{r}-\vec{r}_o)\ \rho_o ds_o d\phi_o \\ &- i\nabla \int_0^l\int_0^{2\pi} \vec{K}(\phi_o,s_o) \bullet \nabla_o G(\vec{r}-\vec{r}_o)\ \rho_o ds_o d\phi_o \end{aligned} \tag{51}$$

$$\begin{aligned} 4\pi\vec{H}^{sc}(\vec{r}) = &\ k\nabla \times \int_0^l\int_0^{2\pi} \vec{K}(\phi_o,s_o)\ G(\vec{r}-\vec{r}_o)\ \rho_o ds_o d\phi_o \\ &+ ik^2 \int_0^l\int_0^{2\pi} \vec{L}(\phi_o,s_o)\ G(\vec{r}-\vec{r}_o)\ \rho_o ds_o d\phi_o \\ &- i\nabla \int_0^l\int_0^{2\pi} \vec{L}(\phi_o,s_o) \bullet \nabla_o G(\vec{r}-\vec{r}_o)\ \rho_o ds_o d\phi_o \end{aligned} \tag{52}$$

where $\vec{r}_o = (\rho_o, \phi_o, z_o)$ and $\rho_o = \rho_g(s_o)$, $z_o = z_g(s_o)$.

B. THE FAR FIELD

In the far-field, the spherical coordinate system $(r,\ \theta,\ \phi)$ is more convenient. To the lowest order in $\frac{1}{r}$:

$$G\ (\vec{r}-\vec{r}_o) \approx \frac{e^{ikr}}{r}\ e^{-i\ [\ kz_o\cos\theta +\ k\rho_o\sin\theta\cos(\phi-\phi_o)\ ]} \tag{53}$$

and

$$\frac{1}{k}\,\nabla\, G\,(\vec{r}-\vec{r}_o)\,\approx\, i\,\hat{r}\,G\,(\vec{r}-\vec{r}_o) \tag{54}$$

where $\cos\theta = \frac{z}{r}$ and $\sin\theta = \frac{\rho}{r}$. Eq. (51) simplifies to:

$$\begin{aligned}\vec{E}^{sc}(\vec{r}) \approx ik\frac{e^{ikr}}{4\pi r}\int_0^l ds_o\int_0^{2\pi} d\phi_o \rho_o [\hat{\theta}[&K_t \sin\theta_g \cos\theta\cos(\phi-\phi_o) + K_\phi \cos\theta\sin(\phi-\phi_o)\\ &-K_t\cos\theta_g\sin\theta + L_\phi\cos(\phi-\phi_o) - L_t\sin\theta_g\sin(\phi-\phi_o)]\\ &+\hat{\phi}[K_\phi\cos(\phi-\phi_o) - K_t\sin\theta_g\sin(\phi-\phi_o)\\ &-L_t\sin\theta_g\cos\theta\cos(\phi-\phi_o) + L_t\cos\theta_g\sin\theta\\ &-L_\phi\cos\theta\sin(\phi-\phi_o)]\ ]e^{-i[kz_o\cos\theta + k\rho_o\sin\theta\cos(\phi-\phi_o)]}\end{aligned} \tag{55}$$

The Fourier expansion of eq. (55) is:

$$\begin{aligned}\vec{E}^{sc}_n(\vec{r}) = ik\frac{e^{ikr}}{2r}(-i)^n\int_0^l ds_o\ \rho_o e^{-ikz_o\cos\theta}\ \{\hat{\theta}\ &[iJ_n'(k\rho_o\sin\theta)(\cos\theta\sin\theta_g K_{t,n} + L_{\phi,n})\\ &+J_n(k\rho_o\sin\theta)(\frac{n\cos\theta}{k\rho_o\sin\theta}K_{\phi,n} - \cos\theta_g\sin\theta K_{t,n} - \frac{n\sin\theta_g}{k\rho_o\sin\theta}L_{t,n})]\\ &+\hat{\phi}[iJ_n'(k\rho_o\sin\theta)(K_{\phi,n} - \cos\theta\sin\theta_g L_{t,n})\\ &+J_n(k\rho_o\sin\theta)(\cos\theta_g\sin\theta L_{t,n} - \frac{n\sin\theta_g}{k\rho_o\sin\theta}K_{t,n} - \frac{n\cos\theta}{k\rho_o\sin\theta}L_{\phi,n})]\}\end{aligned} \tag{56}$$

where $J_n(x)$ is the Bessel function of the first kind of order n, $J'_n(x)$ is the derivative of $J_n(x)$ with respect to x.

For the scattered field in the $\theta = 0$ direction, eq. (55) is further simplified and transformed to:

$$\vec{E}^{sc}(\vec{r}) = k\,\frac{e^{ikr}}{4r}\int_0^l ds\rho_o e^{-ikz_o}\{[(K_{\phi,1}+iL_{\phi,1})+i(K_{t,1}+iL_{t,1})\sin\theta_g](\hat{x}+i\hat{y})$$
$$-[(K_{\phi,-1}-iL_{\phi,-1})-i(K_{t,-1}-iL_{t,-1})\sin\theta_g](\hat{x}-i\hat{y})\} \quad \textbf{(57)}$$

The backscattering cross section is given by:

$$\sigma_B = \lim_{r\to\infty} 4\pi r^2\,\frac{|\vec{E}^{sc}|^2}{|\vec{E}^{inc}|^2} \quad \textbf{(58)}$$

For an incident plane wave directed along axis of the shell with $|\vec{E}^{inc}|$ normalized to unity, σ_B becomes:

$$\sigma_B = \frac{\pi k^2}{4}\,|\int_0^l ds\rho_o e^{-ikz_o}\{[(K_{\phi,1}+iL_{\phi,1})+i(K_{t,1}+iL_{t,1})\sin\theta_g](\hat{x}+i\hat{y})$$
$$-[(K_{\phi,-1}-iL_{\phi,-1})-i(K_{t,-1}-iL_{t,-1})\sin\theta_g](\hat{x}-i\hat{y})\}|^2 \quad \textbf{(59)}$$

III. ZERO ON-AXIS BACKSCATTERING CROSS SECTION

A. INTEGRODIFFERENTIAL EQUATIONS FOR THE SURFACE CURRENTS

The impedance boundary condition links the tangential components of the total electric field $\vec{E}$ to the tangential components of the total magnetic field $\vec{H}$ on a surface through its surface impedance, which is a function of the electromagnetic properties of the material of the surface. This condition is:

$$\hat{n}^{\pm} \times (\vec{E}^{\pm} \times \vec{n}^{\pm}) = Z^{\pm} (\hat{n}^{\pm} \times \vec{H}^{\pm}) \quad \textbf{(60)}$$

where $\vec{E}^{\pm} = \vec{E}^{sc\pm} + \vec{E}^{inc\pm}$ and $\vec{H}^{\pm} = \vec{H}^{sc\pm} + \vec{H}^{inc\pm}$ are the total $\vec{E}$ and $\vec{H}$ fields and Z^{+} and Z^{-} are the normalized surface impedances on S^{+} and S^{-} respectively. Since only the tangential components of $\vec{E}^{\pm}$ and $\vec{H}^{\pm}$ are involved in eq. (60), we shall use $\vec{E}^{sc\pm}_{\tan}$, $\vec{H}^{sc\pm}_{\tan}$, $\vec{E}^{inc\pm}_{\tan}$, $\vec{H}^{inc\pm}_{\tan}$ to denote the tangential components of the scattered and incident fields on the surfaces S^{+} and S^{-}.

In terms of surface electric and magnetic current distributions eq. (60) becomes:

$$\pm\, \hat{n}^{+} \times \vec{L}^{\pm} = Z^{\pm} \vec{K}^{\pm} \quad \textbf{(61)}$$

For anisotropic materials, it is more convenient to utilize matrix notations. In eq. (61) the vectors $\vec{K}^{\pm}$ and $\vec{L}^{\pm}$ can be considered as two-element column vectors, with each of the ϕ-component designated as element 1 and the t-component as element 2. Z^{+} and Z^{-} are then two-by-two matrices.

The cross product of the unit outward normal $\hat{n}^{+}$ with such a two-element column

vector, which is composed of ϕ- and t-components, can be represented by the following matrix multiplication:

$$\hat{n}^+ \times \vec{L}^\pm = V \vec{L}^\pm \tag{62}$$

where $V = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $V^T = -V$ and $V^2 = -I$. Hence, in terms of the matrix V, the impedance boundary condition, eq. (61), can be written as:

$$\begin{aligned} Z^+ \vec{K}^+ &= V \vec{L}^+ \\ Z^- \vec{K}^- &= -V \vec{L}^- \end{aligned} \tag{63}$$

It should be noted that $\vec{K}^\pm = \hat{n}^\pm \times \vec{H}^\pm$ and $\vec{L}^\pm = \vec{E}^\pm \times \hat{n}^\pm$ involves $\vec{E}^{sc\pm}$ and $\vec{H}^{sc\pm}$ which can be computed from $\vec{K} = \vec{K}^+ + \vec{K}^-$ and $\vec{L} = \vec{L}^+ + \vec{L}^-$ via eqs. (51) and (52). Since the difference currents $\vec{K}^+ - \vec{K}^-$ and $\vec{L}^+ - \vec{L}^-$ can also be obtained from eqs. (51) and (52), the impedance boundary condition, eq. (63), if written as a relation between the sum and difference currents, becomes a set of integrodifferential equations for the sum currents. Since

$$Z^+ \vec{K}^+ = \frac{1}{2} Z^+ [\, \vec{K} + (\vec{K}^+ - \vec{K}^-) \,] \tag{64}$$

$$Z^- \vec{K}^- = \frac{1}{2} Z^- [\, \vec{K} - (\vec{K}^+ - \vec{K}^-) \,] \tag{65}$$

eq. (63) can be written as:

$$V \vec{L}^+ = \frac{1}{2} Z^+ [\, \vec{K} + (\vec{K}^+ - \vec{K}^-) \,] \tag{66}$$

$$V \vec{L}^- = -\frac{1}{2} Z^- [\vec{K} - (\vec{K}^+ - \vec{K}^-)] \qquad \textbf{(67)}$$

Define Z and Δ as:

$$Z = \frac{1}{2} (Z^+ + Z^-) \qquad \textbf{(68)}$$

$$\Delta = \frac{1}{2} (Z^+ - Z^-) \qquad \textbf{(69)}$$

By subtracting eq. (67) from eq. (66) and substituting Z and Δ in the result, we have

$$V (\vec{L}^+ - \vec{L}^-) = Z \vec{K} + \Delta (\vec{K}^+ - \vec{K}^-) \qquad \textbf{(70)}$$

By adding the two equations together, we have

$$V \vec{L} = Z (\vec{K}^+ - \vec{K}^-) + \Delta \vec{K} \qquad \textbf{(71)}$$

Assume that Z^{-1} exists, we obtain:

$$\vec{K}^+ - \vec{K}^- = Z^{-1} V \vec{L} - Z^{-1} \Delta \vec{K} \qquad \textbf{(72)}$$

Substituting eq. (72) this into eq. (71), we have:

$$\vec{L}^+ - \vec{L}^- = -V [Z - \Delta Z^{-1} \Delta] \vec{K} - V \Delta Z^{-1} V \vec{L} \qquad \textbf{(73)}$$

Since

$$\pm V L^{\pm} = \hat{n}^{\pm} \times (\vec{E}^{\pm} \times \hat{n}^{\pm}) = \vec{E}^{\pm}_{\tan} = (\vec{E}^{sc\pm}_{\tan} + \vec{E}^{inc\pm}_{\tan}) \qquad \textbf{(74)}$$

and

$$\pm VK^{\pm} = \hat{n}^{\pm} \times (\hat{n}^{\pm} \times \vec{H}^{\pm}) = -\vec{H}^{\pm}_{\tan} = -(\vec{H}^{sc\pm}_{\tan} + \vec{H}^{inc\pm}_{\tan}) \tag{75}$$

Eqs. (72) and (73) can be written, with the understanding that $\vec{E}^{inc+}_{\tan} + \vec{E}^{inc-}_{\tan} = 2\,\vec{E}^{inc}_{\tan}$ and $\vec{H}^{inc+}_{\tan} + \vec{H}^{inc-}_{\tan} = 2\,\vec{H}^{inc}_{\tan}$, as:

$$\vec{E}^{sc+}_{\tan} + \vec{E}^{sc-}_{\tan} + 2\vec{E}^{inc}_{\tan} = [Z - \Delta Z^{-1}\Delta]\vec{K} + \Delta Z^{-1}V\,\vec{L} \tag{76}$$

$$\vec{H}^{sc+}_{\tan} + \vec{H}^{sc-}_{\tan} + 2\vec{H}^{inc}_{\tan} = V\,Z^{-1}\Delta\,\vec{K} - V\,Z^{-1}V\,\vec{L} \tag{77}$$

As $\vec{r}$ approaches $S^{\pm}$ from outside S, the tangential components of $\vec{E}^{sc}(\vec{r})$ and $\vec{H}^{sc}(\vec{r})$ are derived in Appendix C as given by eqs. (51) and (52). They are:

$$\begin{aligned}
E^{sc\pm}_{\phi}(\phi,s) &= \hat{\phi}\cdot\vec{E}^{sc\pm} \\
&= \frac{ik^2}{4\pi}\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[K_\phi\cos(\phi-\phi_o) - K_t\sin(\phi-\phi_o)\sin\theta_g(s_o)]G(\vec{r}-\vec{r}_o) \\
&\quad - \frac{i}{4\pi\rho_g(s)}\frac{\partial}{\partial\phi}\int_0^l\int_0^{2\pi} ds_o d\phi_o [K_\phi\frac{\partial}{\partial\phi_o}G(\vec{r}-\vec{r}_o) + \rho_g(s_o)K_t\frac{\partial}{\partial s_o}G(\vec{r}-\vec{r}_o)] \\
&\quad - \frac{k}{4\pi}\frac{\partial}{\partial s}\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[L_\phi\ \sin(\phi-\phi_o)\cos\theta_g(s) \\
&\quad + L_t\cos(\phi-\phi_o)\cos\theta_g(s)\sin\theta_g(s_o) - L_t\sin\theta_g(s)\cos\theta_g(s_o)]G(\vec{r}-\vec{r}_o) \\
&\quad - \frac{k}{4\pi}\left[\kappa_g(s) - \left.\frac{\partial}{\partial n}\right|_{S^{\pm}}\right]\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[L_t\cos(\phi-\phi_o)\sin\theta_g(s)\sin\theta_g(s_o) \\
&\quad + L_t\cos\theta_g(s)\cos\theta_g(s_o) + L_\phi\sin(\phi-\phi_o)\sin\theta_g(s)]G(\vec{r}-\vec{r}_o)
\end{aligned} \tag{78}$$

$$
\begin{aligned}
E_t^{sc\pm}(\phi,s) &= \hat{t}\cdot\vec{E}^{sc\pm} \\
&= \frac{ik^2}{4\pi}\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[K_\phi \sin(\phi-\phi_o)\sin\theta_g(s) \\
&\quad + K_t\cos(\phi-\phi_o)\sin\theta_g(s)\sin\theta_g(s_o) + K_t\cos\theta_g(s)\cos\theta(s_o)]G(\vec{r}-\vec{r}_o) \\
&\quad - \frac{i}{4\pi}\frac{\partial}{\partial s}\int_0^l\int_0^{2\pi} ds_o d\phi_o [K_\phi \frac{\partial}{\partial\phi_o}G(\vec{r}-\vec{r}_o) + \rho_g(s_o)K_t\frac{\partial}{\partial s_o}G(\vec{r}-\vec{r}_o)] \\
&\quad + \frac{k}{4\pi\rho_g(s)}\frac{\partial}{\partial\phi}\int_0^l\int_0^{2\pi} ds_o d\phi_o\rho_g(s_o)[L_\phi\sin(\phi-\phi_o)\cos\theta_g(s) \\
&\quad - L_t\sin\theta_g(s)\cos\theta_g(s_o) + L_t\cos(\phi-\phi_o)\cos\theta_g(s)\sin\theta_g(s_o)]G(\vec{r}-\vec{r}_o) \\
&\quad - \frac{k}{4\pi}\left[\frac{\cos\theta_g(s)}{\rho_g(s)} + \left.\frac{\partial}{\partial n}\right|_{S^\pm}\right]\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)\ [L_\phi\ \cos(\phi-\phi_o) \\
&\quad - L_t\sin(\phi-\phi_o)\sin\theta_g(s_o)]G(\vec{r}-\vec{r}_o)
\end{aligned}
\tag{79}
$$

where $\left.\frac{\partial}{\partial n}\right|_{S^\pm}$ is the normal derivative taken in the limit as $\vec{r}$ approaches the surface S^+ or S^- respectively from outside of S. It is evaluated as a limiting value in this report and should not be confused with the Fourier index n. Similarly, we have:

$$
\begin{aligned}
H_\phi^{sc\pm}(\phi,s) &= \frac{ik^2}{4\pi}\int_0^l\int_0^{2\pi} ds_o d\phi_o\rho_g(s_o)[L_\phi\cos(\phi-\phi_o) - L_t\sin(\phi-\phi_o)\sin\theta_g(s_o)]G(\vec{r}-\vec{r}_o) \\
&\quad - \frac{i}{4\pi\rho_g(s)}\frac{\partial}{\partial\phi}\int_0^l\int_0^{2\pi} ds_o d\phi_o[L_\phi\frac{\partial}{\partial\phi_o}G(\vec{r}-\vec{r}_o) + \rho_g(s_o)L_t\frac{\partial}{\partial s_o}G(\vec{r}-\vec{r}_o)] \\
&\quad + \frac{k}{4\pi}\frac{\partial}{\partial s}\int_0^l\int_0^{2\pi} ds_o d\phi_o\rho_g(s_o)[K_\phi(\phi_o,s_o)\ \sin(\phi-\phi_o)\cos\theta_g(s) \\
&\quad + K_t\cos(\phi-\phi_o)\cos\theta_g(s)\sin\theta_g(s_o) - K_t(\phi_o,s_o)\sin\theta_g(s)\cos\theta_g(s_o)]G(\vec{r}-\vec{r}_o) \\
&\quad + \frac{k}{4\pi}\left[\kappa_g(s) - \left.\frac{\partial}{\partial n}\right|_{S^\pm}\right]\int_0^l\int_0^{2\pi} ds_o d\phi_o\rho_g(s_o)[K_t\cos(\phi-\phi_o)\sin\theta_g(s)\sin\theta_g(s_o) \\
&\quad + K_t\cos\theta_g(s)\cos\theta_g(s_o) + K_\phi\sin(\phi-\phi_o)\sin\theta_g(s)]G(\vec{r}-\vec{r}_o)
\end{aligned}
\tag{80}
$$

$$
\begin{aligned}
H_t^{sc\pm}(\phi,s) = &\frac{ik^2}{4\pi}\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[L_\phi \sin(\phi-\phi_o)\sin\theta_g(s) \\
&+L_t\cos(\phi-\phi_o)\sin\theta_g(s)\sin\theta_g(s_o)+L_t\cos\theta_g(s)\cos\theta(s_o)]G(\vec{r}-\vec{r}_o) \\
&-\frac{i}{4\pi}\frac{\partial}{\partial s}\int_0^l\int_0^{2\pi} ds_o d\phi_o [L_\phi \frac{\partial}{\partial\phi_o}G(\vec{r}-\vec{r}_o)+\rho_g(s_o)L_t\frac{\partial}{\partial s_o}G(\vec{r}-\vec{r}_o)] \\
&-\frac{k}{4\pi\rho_g(s)}\frac{\partial}{\partial\phi}\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[K_\phi \sin(\phi-\phi_o)\cos\theta_g(s) \\
&-K_t\sin\theta_g(s)\cos\theta_g(s_o)+K_t\cos(\phi-\phi_o)\cos\theta_g(s)\sin\theta_g(s_o)]G(\vec{r}-\vec{r}_o) \\
&+\frac{k}{4\pi}\left[\frac{\cos\theta_g(s)}{\rho_g(s)}+\left.\frac{\partial}{\partial n}\right|_{S^\pm}\right]\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)\ [K_\phi\ \cos(\phi-\phi_o) \\
&-K_t\sin(\phi-\phi_o)\sin\theta_g(s_o)]G(\vec{r}-\vec{r}_o)
\end{aligned}
\tag{81}
$$

As $\vec{r}$ approaches S^+ and S^-, $\rho\to\rho_g(s)$ and $z\to z_g(s)$, the Fourier expansions of eqs. (78) and (79) are:

$$
\begin{aligned}
E_{\phi,n}^{sc\pm}(s) = &\frac{i}{2}\int_o^l ds_o[\frac{k^2}{2}\rho_g(s_o)(G_{n+1}+G_{n-1}) - \frac{n^2}{\rho_g(s)}G_n]\ K_{\phi,n} \\
&+\frac{1}{2}\int_o^l ds_o\rho_g(s_o)[\frac{k^2}{2}\sin\theta_g(s_o)(G_{n+1}-G_{n-1})+\frac{n}{\rho_g(s)}\frac{\partial}{\partial s_o}G_n]\ K_{t,n} \\
&-\frac{ik}{4}\frac{\partial}{\partial s}\int_o^l ds_o\rho_g(s_o)\ \cos\theta_g(s)(G_{n+1}-G_{n-1})\ L_{\phi,n} \\
&-\frac{ik}{4}\left[\kappa_g(s) - \left.\frac{\partial}{\partial n}\right|_{S^\pm}\right]\int_o^l ds_o\rho_g(s_o)\ \sin\theta_g(s)\ (G_{n+1}-G_{n-1})\ L_{\phi,n} \\
&+\frac{k}{2}\frac{\partial}{\partial s}\int_o^l ds_o\rho_g(s_o)[\sin\theta_g(s)\cos\theta_g(s_o)G_n \\
&\qquad -\frac{1}{2}\cos\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1})]L_{t,n} \\
&-\frac{k}{2}\left[\kappa_g(s)-\left.\frac{\partial}{\partial n}\right|_{S^\pm}\right]\int_o^l ds_o\rho_g(s_o)[\cos\theta_g(s)\cos\theta_g(s_o)G_n \\
&\qquad +\frac{1}{2}\sin\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1})]L_{t,n}
\end{aligned}
\tag{82}
$$

$$
\begin{aligned}
E^{sc\pm}_{t,n}(s) \;=\; & -\frac{n}{2}\frac{\partial}{\partial s}\int_o^l ds_o G_n K_{\phi,n} - \frac{k^2}{4}\int_o^l ds_o \rho_g(s_o)\sin\theta_g(s)\,(G_{n+1}-G_{n-1})K_{\phi,n} \\
& - \frac{i}{2}\frac{\partial}{\partial s}\int_o^l ds_o\;\rho_g(s_o)\;[\frac{\partial}{\partial s_o}\;G_n]\;K_{t,n} \\
& + \frac{ik^2}{2}\int_o^l ds_o\;\rho_g(s_o)\;[\cos\theta_g(s)\cos\theta_g(s_o)\,G_n \\
& \qquad\qquad + \frac{1}{2}\sin\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1})]\;K_{t,n} \\
& - \frac{k}{4}\left[\frac{\cos\theta_g(s)}{\rho_g(s)} + \left.\frac{\partial}{\partial n}\right|_{S^\pm}\right]\int_o^l ds_o\;\rho_g(s_o)\;(G_{n+1}+G_{n-1})\;L_{\phi,n} \\
& - \frac{kn\cos\theta_g(s)}{4\rho_g(s)}\int_o^l ds_o\;\rho_g(s_o)\;(G_{n+1}-G_{n-1})\;L_{\phi,n} \\
& + \frac{ik}{4}\left[\frac{\cos\theta_g(s)}{\rho_g(s)} + \left.\frac{\partial}{\partial n}\right|_{S^\pm}\right]\int_o^l ds_o\;\rho_g(s_o)\;\sin\theta_g(s_o)(G_{n+1}-G_{n-1})\;L_{t,n} \\
& + \frac{ikn}{2\rho_g(s)}\int_o^l ds_o\;\rho_g(s_o)\;[\frac{1}{2}\;\cos\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1}) \\
& \qquad\qquad - \sin\theta_g(s)\cos\theta_g(s_o)\;G_n]\;L_{t,n}
\end{aligned}
\qquad \textbf{(83)}
$$

Eqs. (80) and (81) lead to similar forms for $H^{sc\pm}_{\phi,n}$ and $H^{sc\pm}_{t,n}$ by replacing $\vec{L}_n$ with $-\vec{K}_n$ and $\vec{K}_n$ with $\vec{L}_n$ in eqs. (82) and (83). Therefore,

$$
\vec{E}^{sc+}_{\tan,n}(s) + \vec{E}^{sc-}_{\tan,n}(s) = -M_n\,\vec{K}_n\,(s) + N_n\,\vec{L}_n\,(s) \qquad \textbf{(84)}
$$

where the elements of the two-by-two matrices M_n and N_n are integrodifferential operators on the elements of the column vectors $\vec{K}_n$, $\vec{L}_n$ defined by:

$$M_{n,11}K_{\phi,n}(s) = -i\int_o^l ds_o[\frac{k^2}{2}\rho_g(s_o)(G_{n+1}+G_{n-1}) - \frac{n^2}{\rho_g(s)}G_n]\,K_{\phi,n}(s_o)$$

$$M_{n,12}K_{t,n}(s) = -\int_o^l ds_o\rho_g(s_o)[\frac{k^2}{2}\sin\theta_g(s_o)(G_{n+1}-G_{n-1}) + \frac{n}{\rho_g(s)}\frac{\partial}{\partial s_o}G_n]\,K_{t,n}(s_o)$$

$$M_{n,21}K_{\phi,n}(s) = n\frac{\partial}{\partial s}\int_o^l ds_o G_n K_{\phi,n}(s_o) + \frac{k^2}{2}\int_o^l ds_o\rho_g(s_o)\sin\theta_g(s)(G_{n+1}-G_{n-1})K_{\phi,n}(s_o)$$

$$\begin{aligned} M_{n,22}K_{t,n}(s) &= i\,\frac{\partial}{\partial s}\int_o^l ds_o\,\rho_g(s_o)\,[\frac{\partial}{\partial s_o}G_n]\,K_{t,n}(s_o) \\ &- \frac{ik^2}{2}\int_o^l ds_o\,\rho_g(s_o)\,[\cos\theta_g(s)\cos\theta(s_o)G_n \\ &\qquad + \frac{1}{2}\sin\theta_g(s)\sin\theta_g(s_o)(G_{n+1} + G_{n-1})]\,K_{t,n}(s_o) \end{aligned} \tag{85}$$

$$\begin{aligned} N_{n,11}L_{\phi,n}(s) &= -\frac{ik}{2}\frac{\partial}{\partial s}\int_o^l ds_o\rho_g(s_o)\cos\theta_g(s)(G_{n+1}-G_{n-1})L_{\phi,n}(s_o) \\ &- \frac{ik}{4}\left[2\kappa_g(s) - \frac{\partial}{\partial n}\Big|_{S^+} - \frac{\partial}{\partial n}\Big|_{S^-}\right]\int_o^l ds_o\rho_g(s_o)\sin\theta_g(s)(G_{n+1}-G_{n-1})L_{\phi,n}(s_o) \\ N_{n,12}L_{t,n}(s) &= k\frac{\partial}{\partial s}\int_o^l ds_o\rho_g(s_o)[\sin\theta_g(s)\cos\theta_g(s_o)G_n \\ &\qquad - \frac{1}{2}\cos\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1})]L_{t,n}(s_o) \\ &- \frac{k}{2}\left[2\kappa_g(s) - \frac{\partial}{\partial n}\Big|_{S^+} - \frac{\partial}{\partial n}\Big|_{S^-}\right]\int_o^l ds_o\rho_g(s_o)[\cos\theta_g(s)\cos\theta_g(s_o)G_n \\ &\qquad + \frac{1}{2}\sin\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1})]L_{t,n}(s_o) \\ N_{n,21}L_{\phi,n}(s) &= -\frac{k}{4}\left[\frac{2\cos\theta_g(s)}{\rho_g(s)} + \frac{\partial}{\partial n}\Big|_{S^-} + \frac{\partial}{\partial n}\Big|_{S^-}\right]\int_o^l ds_o\rho_g(s_o)(G_{n+1}+G_{n-1})L_{\phi,n}(s_o) \\ &- \frac{kn\cos\theta_g(s)}{2\rho_g(s)}\int_o^l ds_o\rho_g(s_o)(G_{n+1}-G_{n-1})L_{\phi,n}(s_o) \\ N_{n,22}L_{t,n}(s) &= \frac{ik}{4}\left[\frac{2\cos\theta_g(s)}{\rho_g(s)} + \frac{\partial}{\partial n}\Big|_{S^+} + \frac{\partial}{\partial n}\Big|_{S^-}\right]\int_o^l ds_o\rho_g(s_o)\sin\theta_g(s_o)(G_{n+1}-G_{n-1})L_{t,n}(s_o) \\ &+ \frac{ikn}{\rho_g(s)}\int_o^l ds_o\rho_g(s_o)[\frac{1}{2}\cos\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1}) \\ &\qquad - \sin\theta_g(s)\cos\theta_g(s_o)G_n]L_{t,n}(s_o) \end{aligned} \tag{86}$$

Similarly, we can write:

$$\vec{H}^{sc+}_{\tan,n}(s) + \vec{H}^{sc-}_{\tan,n}(s) = -N_n\,\vec{K}_n\,(s) - M_n\,\vec{L}_n\,(s) \tag{87}$$

Combining these with Fourier components of eqs. (76) and (77), we obtain the equation for the sum currents as:

$$\left\{\begin{bmatrix} M_n & -N_n \\ N_n & M_n \end{bmatrix} + \begin{bmatrix} Z-\Delta Z^{-1}\Delta & \Delta Z^{-1}V \\ VZ^{-1}\Delta & -VZ^{-1}V \end{bmatrix}\right\}\begin{bmatrix} \vec{K}_n \\ \vec{L}_n \end{bmatrix} = 2\begin{bmatrix} \vec{E}^{\,inc}_{\tan,n} \\ \vec{H}^{\,inc}_{\tan,n} \end{bmatrix} \tag{88}$$

Note that given the incident wave on the shell, this is a system of linear integrodifferential equations of the second kind for the unknown currents $\vec{K}_n$, $\vec{L}_n$. It is assumed that unique solutions to eq. (88) exist for all *n*.

B. ZERO ON-AXIS BACKSCATTERING

Since the shell is rotationally symmetric, a wave incident along the axis of the shell (the z-axis) can be assumed to be linearly polarized in the y-direction without loss of generality (see Figure 2). Let the fields on the surface of the shell be:

$$\begin{aligned} \vec{E}^{\,inc}(\phi,s) &= \hat{y}\;e^{-ikz_g} \\ \vec{H}^{\,inc}(\phi,s) &= \hat{x}\;e^{-ikz_g} \end{aligned} \tag{89}$$

which represents a plane wave propagating in the *-z* direction. Because $\hat{y} = \cos\phi\,\hat{\phi} + \sin\phi\,\hat{\rho}$ and $\hat{\rho} = \sin\theta_o\,\hat{t} + \cos\theta_o\,\hat{n}+$, then they become

$$\vec{E}^{\,inc}(\phi,s) = [\ \cos\phi\hat{\phi} + \sin\phi\ (\sin\theta_g\hat{t} + \cos\theta_g\hat{n}^{+})\ ]\ e^{-ikz_g(s)}$$
$$\vec{H}^{\,inc}(\phi,s) = [-\sin\phi\hat{\phi} + \cos\phi\ (\sin\theta_g\hat{t} + \cos\theta_g\hat{n}^{+})\ ]\ e^{-ikz_g(s)} \tag{90}$$

The tangential components for the incident fields on the surface of the shell are expressed as:

$$\vec{E}^{\,inc}_{\tan} = \begin{bmatrix} E_\phi^{inc} \\ E_t^{inc} \end{bmatrix} = \begin{bmatrix} \cos\phi \\ \sin\phi\ \sin\theta_g \end{bmatrix} e^{-ikz_g}$$
$$\vec{H}^{\,inc}_{\tan} = \begin{bmatrix} H_\phi^{inc} \\ H_t^{inc} \end{bmatrix} = \begin{bmatrix} -\sin\phi \\ \cos\phi\ \sin\theta_g \end{bmatrix} e^{-ikz_g} \tag{91}$$

The Fourier coefficients are nonzero only for $n = \pm 1$:

$$\left\{ \begin{array}{lllll} E_{\phi,1}^{inc} & = & \frac{1}{2}e^{-ikz_g} & = & E_{\phi,-1}^{inc} \\ E_{t,1}^{inc} & = & -\frac{i}{2}\sin\theta_g e^{-ikz_g} & = & -\ E_{t,-1}^{inc} \\ H_{\phi,1}^{inc} & = & \frac{i}{2}e^{-ikz_g} & = & -\ H_{\phi,-1}^{inc} \\ H_{t,1}^{inc} & = & \frac{1}{2}\sin\theta_g e^{-ikz_g} & = & H_{t,-1}^{inc} \end{array} \right. \tag{92}$$

Because only the $n = \pm 1$ terms in the incident field are nonzero, only $\vec{K}_{\pm 1}$ and $\vec{L}_{\pm 1}$ can exist. Therefore, only the $n = \pm 1$ components of eq. (88) have to be considered:

$$\left\{ \begin{bmatrix} M_{\pm 1} & -N_{\pm 1} \\ N_{\pm 1} & M_{\pm 1} \end{bmatrix} + \begin{bmatrix} Z-\Delta Z^{-1}\Delta & \Delta Z^{-1}V \\ VZ^{-1}\Delta & -VZ^{-1}V \end{bmatrix} \right\} \begin{bmatrix} \vec{K}_{\pm 1} \\ \vec{L}_{\pm 1} \end{bmatrix} = 2 \begin{bmatrix} \vec{E}^{\,inc}_{\tan,\pm 1} \\ \vec{H}^{\,inc}_{\tan,\pm 1} \end{bmatrix} \tag{93}$$

The incident fields in eq. (93) can be written in the form

$$\begin{bmatrix} \vec{E}^{\,inc}_{\tan,\pm 1} \\ \vec{H}^{\,inc}_{\tan,\pm 1} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ \mp i \sin\theta_g \\ \pm i \\ \sin\theta_g \end{bmatrix} e^{-ikz_g} \tag{94}$$

which satisfies the following equation:

$$\begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \begin{bmatrix} \vec{E}^{\,inc}_{\tan,\pm 1} \\ \vec{H}^{\,inc}_{\tan,\pm 1} \end{bmatrix} = \pm \begin{bmatrix} \vec{E}^{\,inc}_{\tan,\pm 1} \\ \vec{H}^{\,inc}_{\tan,\pm 1} \end{bmatrix} \tag{95}$$

or, equivalently,

$$\left\{ 1 \mp \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \right\} \begin{bmatrix} \vec{E}^{\,inc}_{\tan,\pm 1} \\ \vec{H}^{\,inc}_{\tan,\pm 1} \end{bmatrix} = 0 \tag{96}$$

Therefore, if both sides of eq. (93) are multiplied with the factor $\left\{ 1 \mp \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \right\}$, then the right hand side of eq. (93) is equal to zero due to eq. (96):

$$\begin{aligned} &\left\{ 1 \mp \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \right\} \left\{ \begin{bmatrix} M_{\pm 1} & -N_{\pm 1} \\ N_{\pm 1} & M_{\pm 1} \end{bmatrix} + \begin{bmatrix} Z - \Delta Z^{-1}\Delta & \Delta Z^{-1} V \\ V Z^{-1} \Delta & -V Z^{-1} V \end{bmatrix} \right\} \begin{bmatrix} \vec{K}_{\pm 1} \\ \vec{L}_{\pm 1} \end{bmatrix} \\ &= 2 \left\{ 1 \mp \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \right\} \begin{bmatrix} \vec{E}^{\,inc}_{\tan,\pm 1} \\ \vec{H}^{\,inc}_{\tan,\pm 1} \end{bmatrix} \\ &= 0 \end{aligned} \tag{97}$$

From eqs. (85) and (86)

$$\begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \begin{bmatrix} M_n & -N_n \\ N_n & M_n \end{bmatrix} = \begin{bmatrix} M_n & -N_n \\ N_n & M_n \end{bmatrix} \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \qquad \textbf{(98)}$$

Therefore, if the matrix $\begin{bmatrix} Z-\Delta\, Z^{-1}\Delta & \Delta\, Z^{-1}V \\ V\, Z^{-1}\Delta & -V\, Z^{-1}V \end{bmatrix}$ also commutes with $\begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix}$, eq. (97) becomes:

$$\left\{ \begin{bmatrix} M_{\pm 1} & -N_{\pm 1} \\ N_{\pm 1} & M_{\pm 1} \end{bmatrix} + \begin{bmatrix} Z-\Delta Z^{-1}\Delta & \Delta Z^{-1}V \\ V\, Z^{-1}\Delta & -V\, Z^{-1}V \end{bmatrix} \right\} \left\{ 1 \mp \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \right\} \begin{bmatrix} \vec{K}_{\pm 1} \\ \vec{L}_{\pm 1} \end{bmatrix} = \mathbf{0} \qquad \textbf{(99)}$$

The existence and uniqueness of solutions to eq. (97) implies that

$$\left\{ 1 \mp \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix} \right\} \begin{bmatrix} \vec{K}_{\pm 1} \\ \vec{L}_{\pm 1} \end{bmatrix} = \mathbf{0} \qquad \textbf{(100)}$$

or, equivalently

$$\vec{K}_{\pm 1} \pm i\vec{L}_{\pm 1} = 0 \qquad \textbf{(101)}$$

Substituting this result into eq. (59), we conclude that $\sigma_B = 0$ if the matrix $\begin{bmatrix} Z-\Delta Z^{-1}\Delta & \Delta Z^{-1}V \\ V\, Z^{-1}\Delta & -V\, Z^{-1}V \end{bmatrix}$ commutes with $\begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix}$. This is a sufficient condition to eliminate on-axis backscattering.

IV. IMPEDANCE MATRICES FOR ZERO ON-AXIS BACKSCATTERING

A. ELEMENTS OF Z AND Δ

It can be verified that the matrix $\begin{bmatrix} Z-\Delta Z^{-1}\Delta & \Delta Z^{-1}V \\ V\,Z^{-1}\Delta & -V\,Z^{-1}V \end{bmatrix}$ commutes with $\begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix}$, if and only if:

$$V\,Z^{-1}\Delta = -\Delta\;Z^{-1}V \tag{102}$$

$$Z + V\,Z^{-1}V \;= \Delta Z^{-1}\Delta \tag{103}$$

where both Z and Δ are 2×2 matrices. Under the assumption that the inverse of Z exists, we analyze eqs. (102) and (103) as follows:

Because the determinant of the two sides of the eq. (102) has opposite signs, we get $\det Z^{-1}\det\Delta \;=\; 0$. Since Z is assumed to be invertible, $\det\Delta \;=\; 0$.

Because of the identity:

$$Z^{-1} \;=\; -\;\frac{1}{\det Z}V\,Z^{T}V \tag{104}$$

and, by multiplying V to both sides of eq. (102):

$$Z^{-1}\Delta \;=\; V\,\Delta\;Z^{-1}V \tag{105}$$

eq. (103) can be transformed to

$$
\begin{aligned}
Z &= \Delta(V\Delta Z^{-1}V) - V Z^{-1}V \\
&= -\Delta V\Delta V\,(V Z^{-1}V) - V Z^{-1}V \\
&= \frac{1}{\det Z}[1+(\Delta V)^2]Z^T
\end{aligned}
\tag{106}
$$

where

$$
(\Delta V)^2 = \Delta V\Delta V = -\begin{bmatrix} \Delta_{11}\Delta_{22}-\Delta_{12}^2 & 0 \\ 0 & \Delta_{11}\Delta_{22}-\Delta_{21}^2 \end{bmatrix} = \begin{bmatrix} \Delta_{12}(\Delta_{12}-\Delta_{21}) & 0 \\ 0 & -\Delta_{21}(\Delta_{12}-\Delta_{21}) \end{bmatrix} \tag{107}
$$

In the last equation, we utilized the fact that $\Delta_{11}\Delta_{22} = \Delta_{12}\Delta_{21}$ because $\det\Delta = 0$. Substituting eq. (107) into eq. (106) we conclude that:

1. If $Z_{11} \neq 0$ or $Z_{22} \neq 0$, then

$$
1 + \Delta_{12}(\Delta_{12}-\Delta_{21}) = \det Z \tag{108}
$$

$$
1 - \Delta_{21}(\Delta_{12}-\Delta_{21}) = \det Z \tag{109}
$$

$$
Z = Z^T \tag{110}
$$

2. If $Z_{11} = Z_{22} = 0$, then $\det Z = -Z_{12}Z_{21} \neq 0$

$$
1 + \Delta_{12}(\Delta_{12}-\Delta_{21}) = -Z_{12}^2 \tag{111}
$$

$$
1 - \Delta_{21}(\Delta_{12}-\Delta_{21}) = -Z_{21}^2 \tag{112}
$$

On the other hand, substituting eq. (104) into eq. (102) yields:

$$Z_{11}(\Delta_{12} - \Delta_{21}) = (Z_{12} - Z_{21})\,\Delta_{11} \tag{113}$$

$$Z_{22}(\Delta_{12} - \Delta_{21}) = (Z_{12} - Z_{21})\,\Delta_{22} \tag{114}$$

$$Z_{11}\Delta_{22} + Z_{22}\Delta_{11} = 2Z_{21}\Delta_{12} = 2Z_{12}\Delta_{21} \tag{115}$$

For Case 1, Z_{11} and Z_{22} are not both zero and Z is symmetric. Since $Z_{12} = Z_{21}$, eq. (113) or (114) requires that $\Delta_{12} = \Delta_{21}$. Therefore Δ is also symmetric. Eqs. (108) and (109) both requires that $\det Z = 1$. Multiply eq. (115) with either Δ_{11} or Δ_{22} and replace $\Delta_{11}\Delta_{22}$ with Δ_{12}^2, we find either

$$Z_{22}\Delta_{11}^2 - 2Z_{12}\Delta_{11}\Delta_{12} + Z_{11}\Delta_{12}^2 = 0 \tag{116}$$

or

$$Z_{11}\Delta_{22}^2 - 2Z_{12}\Delta_{22}\Delta_{12} + Z_{22}\Delta_{12}^2 = 0 \tag{117}$$

Eq. (116) can be solved for Δ_{11} in terms of Δ_{12}:

$$\Delta_{11} = \frac{Z_{12} \pm \sqrt{Z_{12}^2 - Z_{11}Z_{22}}}{Z_{22}}\,\Delta_{12} = \frac{Z_{12} \pm i}{Z_{22}}\,\Delta_{12}$$

Multiplying Δ_{22} to both sides and replacing $\Delta_{11}\Delta_{22}$ with Δ_{12}^2 again result in:

$$\Delta_{12} = \frac{Z_{12} \pm i}{Z_{22}}\,\Delta_{22}$$

Hence both Δ_{11} and Δ_{12} can be given in terms of Δ_{22} as:

$$\begin{aligned} \Delta_{11} &= \left(\frac{Z_{12} \pm i}{Z_{22}}\right)^2 \Delta_{22} \\ \Delta_{12} &= \left(\frac{Z_{12} \pm i}{Z_{22}}\right) \Delta_{22} \end{aligned} \qquad \textbf{(118)}$$

Similarly, from eq. (117):

$$\begin{aligned} \Delta_{22} &= \left(\frac{Z_{12} \mp i}{Z_{11}}\right)^2 \Delta_{11} \\ \Delta_{12} &= \left(\frac{Z_{12} \mp i}{Z_{11}}\right) \Delta_{11} \end{aligned} \qquad \textbf{(119)}$$

Eqs. (118) and (119) are equivalent and both include the situation $\Delta = 0$. The particular one to use is a matter of convenience, especially when either $Z_{11} = 0$ or $Z_{22} = 0$. Note that $Z_{12} = Z_{21} = \pm i$ if $Z_{11}Z_{22} = 0$ because $\det Z = 1$.

For Case 2, $Z_{11} = Z_{22} = 0$ and $Z_{12}Z_{21} \neq 0$. Eq. (115) requires that $\Delta_{12} = \Delta_{21} = 0$, therefore $\Delta_{11}\,\Delta_{22} = \Delta_{12}\,\Delta_{21} = 0$. Eqs. (111) and (112) reduce to:

$$Z_{12}^2 = Z_{21}^2 = -1 \qquad \textbf{(120)}$$

If $Z_{12} = Z_{21} = \pm i$, then there is no further requirement on Δ except that at least one of Δ_{11} or Δ_{22} vanishes. If $Z_{12} = -Z_{21} = \pm i$, then eqs. (113) and (114) requires that $\Delta_{11} = \Delta_{22} = 0$. Hence

$$Z^{+} = Z^{-} = Z = \begin{bmatrix} 0 & \pm i \\ \mp i & 0 \end{bmatrix} \quad \textbf{(121)}$$

This is the only situation when $\det Z \neq 1$.

B. Z^{+} AND Z^{-}

The impedance matrices for zero on-axis backscattering specified in Sec. A above can be stated in terms of Z^{+} and Z^{-} directly. When Z^{+} and Z^{-} are skew-symmetric, the only possibility is eq. (121). When Z^{+} and Z^{-} are symmetric, the requirements are:

$$\det Z^{+} = 1 \quad \textbf{(122)}$$

$$\det Z^{-} = 1 \quad \textbf{(123)}$$

$$\det(Z^{+} - Z^{-}) = 0 \quad \textbf{(124)}$$

Eq. (124) is just $\det\Delta = 0$. To show that conditions (122) through (124) are equivalent to eqs. (108) to (115), note that eqs. (110), (113), and (114) are satisfied trivially. Eqs. (108), (109), (111) and (112) become:

$$\det Z = 1 \quad \textbf{(125)}$$

and eq. (115) becomes:

$$Z_{11}\Delta_{22} + Z_{22}\Delta_{11} = 2Z_{12}\Delta_{12} \tag{126}$$

Since $Z^+ = Z + \Delta$ and $Z^- = Z - \Delta$, eqs. (122) and (123) are:

$$(Z_{11} + \Delta_{11})\,(Z_{22} + \Delta_{22}) - (Z_{12} + \Delta_{12})^2 = 1 \tag{127}$$

$$(Z_{11} - \Delta_{11})\,(Z_{22} - \Delta_{22}) - (Z_{12} - \Delta_{12})^2 = 1 \tag{128}$$

Except for an overall factor of 2, the difference of eqs. (127) and (128) is eq. (126); the sum of these two is:

$$Z_{11}Z_{22} + \Delta_{11}\Delta_{22} - Z_{12}^2 - \Delta_{12}^2 = 1 \tag{129}$$

which, when combined with $\det\Delta = 0$, is just $\det Z = 1$. Together with eq. (126),

$$\begin{aligned}\det(Z + \alpha\Delta) &= \det Z + \alpha \det\begin{bmatrix} Z_{11} & \Delta_{12} \\ Z_{12} & \Delta_{22} \end{bmatrix} + \alpha \det\begin{bmatrix} \Delta_{11} & Z_{12} \\ \Delta_{12} & Z_{22} \end{bmatrix} + \alpha^2 \det\Delta \\ &= \det Z = 1\end{aligned}$$

for any α. Therefore, with $\alpha = \pm 1$, $\det Z^+ = \det Z^- = 1$. Hence, with symmetric Z^+ and Z^-, conditions (122) to (124) specify completely the impedance matrices which eliminate on-axis backscattering.

It should be noted that, if the shell is a closed surface, then the impedance boundary condition closes off the inside of the shell from outside. Therefore, only $\det Z^+ = 1$ is required to eliminate on-axis backscattering. This is an extension of Weston's work [7] to anisotropic impedance coated body of revolution.

V. CONCLUSIONS

A sufficient condition for eliminating the on-axis backscattering of an anisotropic impedance coated shell of revolution has been deduced. The outside and inside normalized surface impedances Z^+ and Z^- with which this sufficient condition can be satisfied have all been found. One exceptional situation is when the impedance matrices are equal and skew-symmetric with their determinants equal to -1. All other cases require that the two matrices be symmetric, their determinants be unity, and the determinant of their difference be zero.

Based on the duality of Maxwell equations, Weston [7] argued that an object coated with a unit normalized surface impedance will have zero on-axis backscattering if the object is invariant under a $90°$ rotation around its axis of symmetry. It is clear from Chapter III of this report that the duality property is crucial in determining the sufficient condition for zero on-axis backscattering. Results of this work therefore should not be limited to shells of revolution only. This aspect of the problem is being completed and will be published shortly.

The shell under consideration can be a closed one. A body of revolution coated on the outside with an anisotropic surface impedance Z^+ can be modeled as a shell having $Z^+ = Z^-$. Since the impedance boundary condition separates completely the exterior of the body from its interior, only the conditions on Z^+ need to apply, i.e., Z^+ must be either symmetric or skew-symmetric, with $\det Z^+ = \pm 1$. This is an extension of Weston's result to anisotropically coated bodies.

It should be noted that a properly shaped and coated body can always be carefully deformed into a shell without generating any on-axis backscattering. This procedure yields

the condition $Z^+ = Z^-$. By beginning with a shell, we are able to relax this condition to $\det(Z^+ - Z^-) = 0$ for the symmetric case.

Results of this work make available a wide class of models which must have zero on-axis backscattering cross section. All general purpose numerical codes for computing the scattering cross sections of anisotropic impedance coated objects should be checked for their accuracy against a selected group of such models. Such comparisons should provide indications of an error bound of the particular algorithm.

APPENDIX A: VECTOR CALCULUS ON A SURFACE OF REVOLUTION

Vector calculus near a surface of revolution will be presented in this appendix. In particular, the operations of the divergence and curl on a vector field will be given explicitly. The derivation will be based on vector calculus in the familiar cylindrical coordinates (ρ, ϕ, z).

The axis of revolution is taken to be the z- axis. In the half plane of a fixed ϕ, the generating curve of the surface is parameterized in its arc length s with the functions $z_g(s)$ and $\rho_g(s)$:

$$\vec{r}(s) = z_g(s)\hat{z} + \rho_g(s)\hat{\rho} \tag{130}$$

The surface is thus parameterized in terms of (ϕ, s). This generating curve may be closed, open, discontinuous or self-intercept. Nevertheless, at every point in a smooth segment of the generating curve, an outward unit normal to the "outer" surface, denoted by $\hat{n}^+$, can always be defined. The parameter s can be oriented so that the tangent vector $\hat{t}$ to the generating curve satisfies the relation:

$$\hat{n}^+ = \hat{\phi} \times \hat{t} \tag{131}$$

Note that:

$$\hat{t} = \frac{d\vec{r}}{ds} = \cos\theta_g(s)\hat{z} + \sin\theta_g(s)\hat{\rho} \tag{132}$$

where $\cos\theta_g(s) = \dfrac{d}{ds}z_g(s)$, $\sin\theta_g(s) = \dfrac{d}{ds}\rho_g(s)$; $\theta_g(s)$ is the angle measured from $\hat{z}$ to $\hat{t}$,

defined to be a continuous function of s along any smooth segment of the generating curve.

Since

$$\frac{d^2\vec{r}}{ds^2} = (-\sin\theta_g(s)\hat{z} + \cos\theta_g(s)\hat{\rho})\frac{d}{da}\theta_g(s) = \kappa_g(s)\hat{n}^+ \qquad \textbf{(133)}$$

where $\kappa_g(s) = \frac{d}{ds}\theta_g(s)$ is the signed curvature [8] of the generating curve: κ_g is positive when θ_g is increasing with s and $|1/\kappa_g|$ is the radius of curvature of the generating curve.

In the neighborhood of the surface of revolution, the space can be described with the orthogonal coordinates (ϕ, s, n), $n = 0$ being the surface and $n > 0$ going in the $\hat{n}^+$ direction. In relation to the cylindrical coordinate system, ϕ is identical and, for $n \to 0$, the transformation of dz, $d\rho$ into ds, dn is

$$\begin{bmatrix} ds \\ dn \end{bmatrix} = \begin{bmatrix} \cos\theta_g & \sin\theta_g \\ -\sin\theta_g & \cos\theta_g \end{bmatrix} \begin{bmatrix} dz \\ d\rho \end{bmatrix} \qquad \textbf{(134)}$$

The transformation of ds, dn into dz, $d\rho$ follows the same rule as the matrix is unitary. This equation also gives, for $n \to 0$:

$$\begin{aligned} \frac{\partial s}{\partial z} &= \frac{\partial n}{\partial \rho} = \frac{\partial z}{\partial s} = \frac{\partial \rho}{\partial n} = \cos\theta_g \\ \frac{\partial s}{\partial \rho} &= -\frac{\partial n}{\partial z} = \frac{\partial \rho}{\partial s} = -\frac{\partial z}{\partial n} = \sin\theta_g \end{aligned} \qquad \textbf{(135)}$$

For a vector field $\vec{A} = A_\phi\,\hat{\phi} + A_t\,\hat{t} + A_n\,\hat{n}^+$, A_t and A_n transforms from A_z and A_ρ according to eq. (126). On the other hand,

$$\frac{\partial}{\partial z} = \frac{\partial s}{\partial z}\frac{\partial}{\partial s} + \frac{\partial n}{\partial z}\frac{\partial}{\partial n} = \cos\theta_g \frac{\partial}{\partial s} - \sin\theta_g \frac{\partial}{\partial n}$$
$$\frac{\partial}{\partial \rho} = \frac{\partial s}{\partial \rho}\frac{\partial}{\partial s} + \frac{\partial n}{\partial \rho}\frac{\partial}{\partial n} = \sin\theta_g \frac{\partial}{\partial s} + \cos\theta_g \frac{\partial}{\partial n} \tag{136}$$

Substitute the above into the expressions in the cylindrical coordinate system result in the following as $n \to 0$:

$$\begin{aligned}\nabla \cdot A &= \frac{1}{\rho}\frac{\partial}{\partial \phi}A_\phi + \frac{\partial}{\partial z}A_z + \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) \\ &= \frac{1}{\rho_g}[\frac{\partial}{\partial \phi}A_\phi + \frac{\partial}{\partial s}(\rho A_t) + (\frac{\partial}{\partial n}-\kappa_g)(\rho A_n)] \\ &= \frac{1}{\rho_g}\frac{\partial}{\partial \phi}A_\phi + (\frac{\sin\theta_g}{\rho_g} + \frac{\partial}{\partial s})A_t + (\frac{\cos\theta_g}{\rho_g} - \kappa_g + \frac{\partial}{\partial n})A_n\end{aligned} \tag{137}$$

$$(\nabla \times A)_\phi = \frac{\partial}{\partial z}A_\rho - \frac{\partial}{\partial \rho}A_z = \frac{\partial}{\partial s}A_n + (\kappa_g - \frac{\partial}{\partial n})A_t \tag{138}$$

$$\begin{aligned}(\nabla \times A)_t &= \frac{\cos\theta_g}{\rho}[\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{\partial}{\partial \phi}A_\rho] + \sin\theta_g[\frac{1}{\rho}\frac{\partial}{\partial \phi}A_z - \frac{\partial}{\partial z}A_\phi] \\ &= \frac{1}{\rho_g}[\frac{\partial}{\partial n}(\rho A_\phi) - \frac{\partial}{\partial \phi}A_n] \\ &= (\frac{\cos\theta_g}{\rho_g} + \frac{\partial}{\partial n})A_\phi - \frac{1}{\rho_g}\frac{\partial}{\partial \phi}A_n\end{aligned} \tag{139}$$

$$
\begin{aligned}
(\nabla \times A)_n &= -\frac{\sin\theta_g}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\phi) - \frac{\partial}{\partial\phi}A_\rho\right] + \cos\theta_g\left[\frac{1}{\rho}\frac{\partial}{\partial\phi}A_z - \frac{\partial}{\partial z}A_\phi\right] \\
&= \frac{1}{\rho_g}\left[\frac{\partial}{\partial\phi}A_t - \frac{\partial}{\partial s}(\rho A_\phi)\right] \\
&= \frac{1}{\rho_g}\frac{\partial}{\partial\phi}A_t - \left(\frac{\sin\theta_g}{\rho_g} + \frac{\partial}{\partial s}\right)A_\phi
\end{aligned} \tag{140}
$$

As a check, the substitutions $A_t = -A_\theta$, $A_n = A_r$, $\rho_g = r\sin\theta$, $\theta_g = \theta - \frac{\pi}{2}$, $\kappa_g = -\frac{1}{r}$, $\frac{\partial}{\partial s} = -\frac{1}{r}\frac{\partial}{\partial\theta}$, $\frac{\partial}{\partial n} = \frac{\partial}{\partial r}$ result in the correct expressions for $\nabla \cdot \vec{A}$ and $\nabla \times \vec{A}$ in the spherical coordinate system (r, θ, ϕ).

APPENDIX B: END-CAP CONTRIBUTIONS TO THE SCATTERED FIELDS

In eqs. (45) and (46), the surface S, consisting of the four non-overlapping surfaces S_l, S_o, S_1 and S_2 (Fig. 3), can be shrunk toward the shell. So that S may be the surface of constant distance δ from the shell. In this appendix the fact that contributions from the top cap S_l and the bottom cap S_o to the scattered fields are zero in the limit that the thickness of the shell is infinitesimal will be demonstrated.

An example of the geometry of the end cap is shown in Fig. 4. Near the end at $s = l$, define:

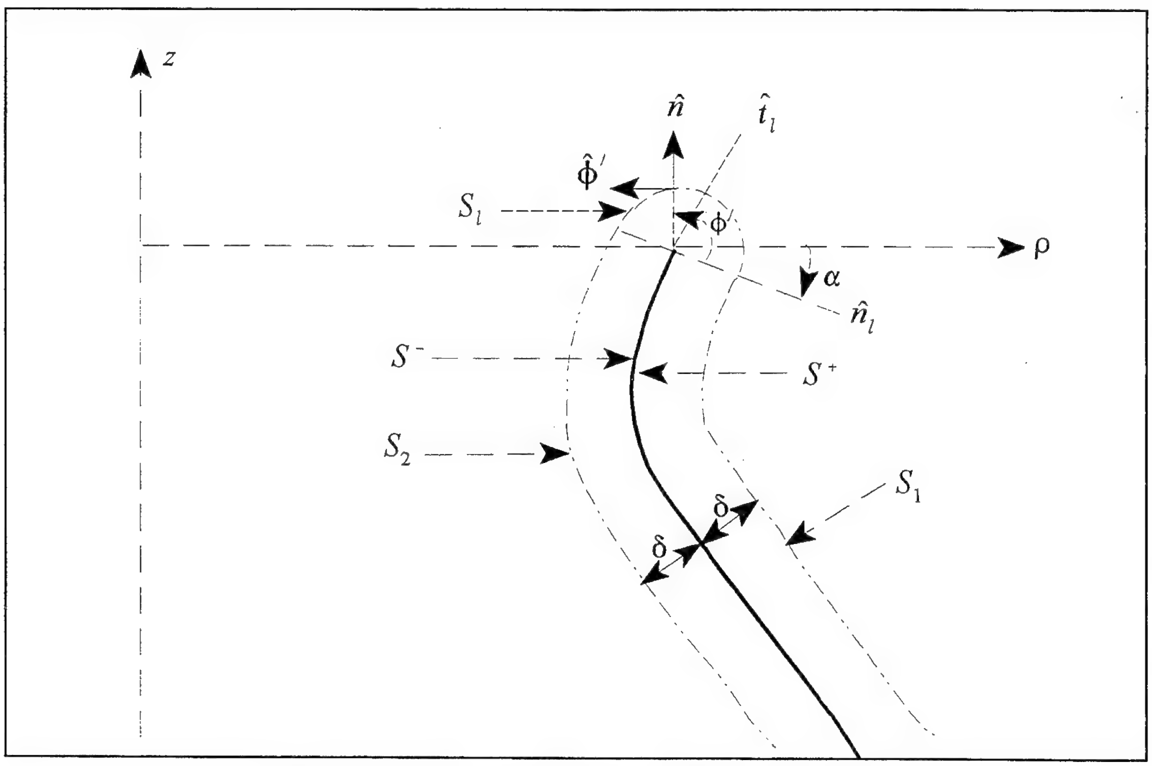

Figure 4 The geometry near an end of the shell.

the normal direction at the end as: $\hat{n}_l = \lim_{s \to l^-} \hat{n}(s)$,

the tangent direction at the end as: $\hat{t}_l = \lim_{s \to l^-} \hat{t}(s)$,

and the tilt angle at the end as: $\alpha = \lim_{s \to l^-} \theta_g(s)$, which is the angle measured from z-axis to $\hat{t}_l$. The cross section of the end-cap at $s = l$ in a half plane of fixed azimuth angle ϕ is a semi-circle with a radius δ. Note that δ, a constant distance distance between S and the shell, is a fixed, positive number. In Fig. 4, the angle ϕ' is measured from the axis $\hat{n}_l$. In terms of the angles ϕ' and α, the end cap S_l can be parameterized in the cylindrical coordinates as $(\rho,\ \phi,\ z)$, where for $0 < \phi' < \pi$,

$$\rho = \rho_g(l) + \delta\cos(\phi' - \alpha)$$

$$z = z_g(l) + \delta\sin(\phi' - \alpha)$$

and $\quad 0 \le \phi < 2\pi$.

Note that on the cap surface, any field vector may be decompossed to the three directional components: ϕ- component, ϕ'-component and n-component, where $\hat{n}$ denotes the outward normal to S_l.

In eq. (45) and (46), the contributions to the scattered fields from surface integrals over the end cap S_l are:

$$\begin{aligned} \vec{A}_1 &= \int_{S_l} [\hat{n} \times \vec{E}(\vec{r}_o)] G(\vec{r}-\vec{r}_o) da_o \\ &= \int_0^{\pi}\int_0^{2\pi} [\hat{n} \times \vec{E}(\vec{r}_o)] G(\vec{r}-\vec{r}_o)\ [\rho_g(l) + \delta\cos(\phi'-\alpha)]\ \delta\ d\phi_o d\phi' \end{aligned} \qquad \textbf{(141)}$$

$$\vec{A}_2 = \int_{S_l} [\hat{n} \times \vec{H}(\vec{r}_o)] G(\vec{r}-\vec{r}_o) \; da_o$$

$$= \int_0^{\pi}\int_0^{2\pi} [\hat{n} \times \vec{H}(\vec{r}_o)] G(\vec{r}-\vec{r}_o) \; [\, \rho_g(l) + \delta\cos(\phi' - \alpha)] \; \delta \; d\phi_o d\phi' \tag{142}$$

$$\psi_1 = \int_{S_l} [\hat{n} \times \vec{H}(\vec{r}_o)] \bullet \nabla_o G(\vec{r}-\vec{r}_o) \; da_o$$

$$= \int_0^{\pi}\int_0^{2\pi} [\hat{n} \times \vec{H}(\vec{r}_o)] \bullet \nabla_o G(\vec{r}-\vec{r}_o) \; [\rho_g(l) + \delta\cos(\phi' - \alpha)] \; \delta \; d\phi_o d\phi' \tag{143}$$

$$\psi_2 = \int_{S_l} [\hat{n} \times \vec{E}(\vec{r}_o)] \bullet \nabla_o G(\vec{r}-\vec{r}_o) \; da_o$$

$$= \int_0^{\pi}\int_0^{2\pi} [\hat{n} \times \vec{E}(\vec{r}_o)] \bullet \nabla_o G(\vec{r}-\vec{r}_o) \; [\rho_g(l) + \delta\cos(\phi' - \alpha)] \; \delta \; d\phi_o d\phi' \tag{144}$$

From the edge conditions [5], on the end cap S_l,

$\hat{n} \times \vec{E} = E_\phi \hat{\phi}' - E_{\phi'} \hat{\phi}$, where $|E_\phi| \le E_1 \delta^{1/2}$ and $|E_{\phi'}| \le E_2 \delta^{-1/2}$,

and $\hat{n} \times \vec{H} = H_\phi \hat{\phi}' - H_{\phi'} \hat{\phi}$, where $|H_\phi| \le H_1 \delta^{1/2}$ and $|H_{\phi'}| \le H_2 \delta^{-1/2}$,

where E_1, E_2, H_1, H_2 are positive constants. Therefore,

$$|\hat{n} \times \vec{E}| \le \delta^{-1/2}\sqrt{E_1^2 \delta^2 + E_2^2}$$
$$|\hat{n} \times \vec{H}| \le \delta^{-1/2}\sqrt{H_1^2 \delta^2 + H_2^2} \tag{145}$$

Note that $\vec{r}$ lies away from the shell. If the distance between $\vec{r}$ and the shell is $\delta_o \ll \frac{1}{k}$, δ can be choose such that $\delta < \frac{1}{2}\delta_0$, so that $\vec{r}$ always lies outside S, with $|\vec{r} - \vec{r}_o| > \frac{\delta_o}{2}$. Therefore $|G(\vec{r}-\vec{r}_o)| \le \frac{2}{k\delta_o} = C_1$ and $\frac{1}{k}|\nabla_o G(\vec{r} - \vec{r}_o)| \le \frac{C}{k^2\delta_o^2} = C_2$ for some positive

constants C, C_1 and C_2

Substitute these into eq. (141) and (142), and write them as:

$$
\begin{aligned}
|\vec{A}_1| &= \left| \int_0^{\pi}\int_0^{2\pi} d\phi_o\, d\phi'\, \delta\, [\rho_g(l) + \delta\cos(\phi' - \alpha)]\ [\hat{n}\times\vec{E}(\vec{r}_o)]\ G(\vec{r}-\vec{r}_o) \right| \\
&\le \int_0^{\pi}\int_0^{2\pi} d\phi_o d\phi' \delta\ |\rho_g(l)+\delta\cos(\phi'-\alpha)|\ |E_\phi\hat{\phi}'-E_\phi\hat{\phi}|\ |G(\vec{r}-\vec{r}_o)| \\
&\le \int_0^{\pi}\int_0^{2\pi} d\phi_o d\phi' \delta\ |\rho_g(l) + \delta\cos(\phi' - \alpha)|\ \left(\delta^{-1/2}\sqrt{E_1^2\delta^2 + E_2^2}\right) C_1 \\
&\le 2\pi^2\left(\rho_g(l) + \delta\right)\left(\delta^{1/2}\sqrt{E_1^2\delta^2 + E_2^2}\right) C_1
\end{aligned}
\tag{146}
$$

Substituting $\hat{n} \times \vec{H}$ into eq. (142), we have:

$$
\begin{aligned}
|\vec{A}_2| &= \left| \int_0^{\pi}\int_0^{2\pi} d\phi_o d\phi'\, \delta\ [\rho_g(l)+\delta\cos(\phi'-\alpha)]\ [\hat{n}\times\vec{H}(\vec{r}_o)]\ G(\vec{r}-\vec{r}_o) \right| \\
&\le 2\pi^2\left(\rho_g(l) + \delta\right)\left(\delta^{1/2}\sqrt{H_1^2\delta^2 + H_2^2}\right) C_1
\end{aligned}
\tag{147}
$$

Similarly, for eqs. (143) and (144), we have:

$$
|\psi_1| \le 2\pi^2\left(\rho_g(l) + \delta\right)\left(\delta^{1/2}\sqrt{E_1^2\delta^2 + E_2^2}\right) C_2 \tag{148}
$$

$$
|\psi_2| \le 2\pi^2\left(\rho_g(l) + \delta\right)\left(\delta^{1/2}\sqrt{H_1^2\delta^2 + H_2^2}\right) C_2 \tag{149}
$$

therefore the limits of $|\vec{A}_1|$, $|\vec{A}_2|$, $|\psi_1|$, $|\psi_2|$ vanishe as $\delta \to 0$.

Hence, in the limit that the thickness of the shell is infinitesimal, the contributions of the integrals over the top cap S_l to the scattered fields are zero. Because the geometry of the bottom cap S_o is similar with the top cap S_l, the results for S_l are also valid for S_o. The

contribution from the bottom cap S_o to the scattered fields is also zero. We conclude that the integrals in eqs. (45) and (46) need to be carried out over S^+ and S^- only.

APPENDIX C: SCATTERED FIELDS NEAR THE SHELL

The object interested in this report is an arbitrarily shaped shell of revolution. Near the surface of relvolution, the vector calculus derived in Appendix A should be used for the operation of the divergence and curl on a vector field. In this appendix the scattered fields near the surfaces S^+ and S^- are going to be derived by using the vector calculus.

From Appendix A, $\hat{t} = \frac{d\vec{r}}{ds} = \hat{z}\cos\theta_g(s) + \hat{\rho}\sin\theta_g(s)$, and $\hat{n}^+ = \hat{\phi} \times t$. The coordinate tranformations between $\vec{r}$ and $\vec{r}_o$ are:

$$
\begin{aligned}
\hat{\phi}_o \cdot \hat{\phi} &= \cos(\phi - \phi_o) \\
\hat{\phi}_o \cdot \hat{t} &= \sin(\phi - \phi_o)\cos\theta_g(s) \\
\hat{\phi}_o \cdot \hat{n}^+ &= \sin(\phi - \phi_o)\sin\theta_g(s) \\
\hat{t}_o \cdot \hat{\phi} &= -\sin(\phi - \phi_o)\sin\theta_g(s_o) \\
\hat{t}_o \cdot \hat{t} &= \cos\theta_g(s)\cos\theta_g(s_o) + \cos(\phi - \phi_o)\sin\theta_g(s)\sin\theta_g(s_o) \\
\hat{t}_o \cdot \hat{n}^+ &= \cos(\phi - \phi_o)\cos\theta_g(s)\sin\theta_g(s_o) - \sin\theta_g(s)\cos\theta_g(s_o)
\end{aligned}
\tag{150}
$$

In eq. (51), the first surface integral can be transformed as:

$$
\begin{aligned}
&- k\int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)\, \vec{L}\, G \\
&= - k\int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)\, (L_t\hat{t}_o + L_\phi\hat{\phi}_o)\, G \\
&= - k\int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)\Big\{\hat{\phi}\,[-L_t\sin(\phi - \phi_o)\sin\theta_g(s_o) + L_\phi\cos(\phi - \phi_o)]\,G \\
&\qquad + \hat{t}\,[L_t\cos\theta_g(s)\cos\theta_g(s_o) + L_t\cos(\phi-\phi_o)\sin\theta_g(s)\sin\theta_g(s_o) \\
&\qquad + L_\phi\sin(\phi-\phi_o)\sin\theta_g(s)]\,G + \hat{n}^+[L_t\cos(\phi - \phi_o)\cos\theta_g(s)\sin\theta_g(s_o) \\
&\qquad - L_t\sin\theta_g(s)\cos\theta_g(s_o) + L_\phi\sin(\phi - \phi_o)\cos\theta_g(s)]\,G\Big\}
\end{aligned}
\tag{151}
$$

the second surface integral can be transformed as:

$$
\begin{aligned}
& ik^2 \int_0^l\int_0^{2\pi} d\phi_o\, ds_o\ \rho_g(s_o)\ \vec{K}\ G \\
&= ik^2 \int_0^l\int_0^{2\pi} d\phi_o\, ds_o\ \rho_g(s_o)\ (K_\phi \hat{\phi}_o + K_t \hat{t}_o)\ G \\
&= ik^2 \int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o) \Big\{ \hat{\phi}[K_\phi \cos(\phi-\phi_o) - K_t \sin(\phi-\phi_o)\sin\theta_g(s_o)]\, G \\
&\qquad + \hat{t}[K_\phi \sin(\phi-\phi_o)\sin\theta_g(s) + K_t \cos\theta_g(s)\cos\theta_g(s_o) \\
&\qquad + K_t \cos(\phi-\phi_o)\sin\theta_g(s)\sin\theta_g(s_o)]\, G + \hat{n}^+[K_\phi \sin(\phi-\phi_o)\cos\theta_g(s) \\
&\qquad - K_t \sin\theta_g(s)\cos\theta_g(s_o) + K_t \cos(\phi-\phi_o)\cos\theta_g(s)\sin\theta_g(s_o)]\, G \Big\}
\end{aligned} \tag{152}
$$

Since $\nabla\psi = (\hat{\phi}\dfrac{1}{\rho_g(s)}\dfrac{\partial}{\partial\phi} + \hat{t}\dfrac{\partial}{\partial s} + \hat{n}^+\dfrac{\partial}{\partial n})\ \psi$, the third surface integral in eq. (51) becomes:

$$
\begin{aligned}
& -i\nabla \int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)\ \ \vec{K} \bullet \nabla_o G \\
&= -i\nabla \int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)(K_\phi \hat{\phi}_o + K_t \hat{t}_o)\cdot\left(\hat{\phi}_o \frac{1}{\rho_g(s_o)}\frac{\partial}{\partial\phi_o} + \hat{t}_o\frac{\partial}{\partial s_o} + \hat{n}_o^+\frac{\partial}{\partial n_o}\right) G \\
&= -i\nabla \int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)\left(\frac{K_\phi}{\rho_g(s_o)}\frac{\partial}{\partial\phi_o}G + K_t\frac{\partial}{\partial s_o}G\right) \\
&= -i\hat{\phi}\frac{1}{\rho_g(s)}\frac{\partial}{\partial\phi}\int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)\left(\frac{K_\phi}{\rho_g(s_o)}\frac{\partial}{\partial\phi_o}G + K_t\frac{\partial}{\partial s_o}G\right) \\
&\quad -i\hat{t}\frac{\partial}{\partial s}\int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)\left(\frac{K_\phi}{\rho_g(s_o)}\frac{\partial}{\partial\phi_o}G + K_t\frac{\partial}{\partial s_o}G\right) \\
&\quad -i\hat{n}\frac{\partial}{\partial n}\int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)\left(\frac{K_\phi}{\rho_g(s_o)}\frac{\partial}{\partial\phi_o}G + K_t\frac{\partial}{\partial s_o}G\right)
\end{aligned} \tag{153}
$$

From eqs (138) to (140), the curl of eq. (151) is:

$$
\begin{aligned}
& - k\nabla\times\int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)\ \vec{L}\ G \\
&= -\ k\hat{\phi}\Bigg\{\frac{\partial}{\partial s}\int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)\,[L_\phi \sin(\phi-\phi_o)\cos\theta_g(s) - L_t \sin\theta_g(s)\cos\theta_g(s_o) \\
&\qquad\qquad + L_t\cos(\phi-\phi_o)\cos\theta_g(s)\sin\theta_g(s_o)]\ G \\
&+ \left(\kappa_g - \frac{\partial}{\partial n}\right)\int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)\,[L_\phi \sin(\phi-\phi_o)\sin\theta_g(s) + L_t\cos\theta_g(s)\cos\theta_g(s_o) \\
&\qquad\qquad + L_t\cos(\phi-\phi_o)\sin\theta_g(s)\sin\theta_g(s_o)]\ G\ \Bigg\} \\
&- k\hat{t}\Bigg\{\left(\frac{\cos\theta_g(s)}{\rho_g(s)} + \frac{\partial}{\partial n}\right)\int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)\,[L_\phi\cos(\phi-\phi_o) \\
&\qquad\qquad - L_t\sin(\phi-\phi_o)\sin\theta_g(s_o)] \\
&- \frac{1}{\rho_g(s)}\frac{\partial}{\partial\phi}\int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)\,[L_\phi \sin(\phi-\phi_o)\cos\theta_g(s) - L_t\sin\theta_g(s)\cos\theta_g(s_o) \\
&\qquad\qquad + L_t\cos(\phi-\phi_o)\cos\theta_g(s)\sin\theta_g(s_o)]\ G\ \Bigg\} \\
&- k\hat{n}\Bigg\{\frac{1}{\rho_g(s)}\frac{\partial}{\partial\phi}\int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)[L_\phi\sin(\phi-\phi_o)\sin\theta_g(s) + L_t\cos\theta_g(s)\cos\theta_g(s_o) \\
&\qquad\qquad + L_t\cos(\phi-\phi_o)\sin\theta_g(s)\sin\theta_g(s_o)]G \\
&- \left(\frac{\sin\theta_g(s)}{\rho_g(s)} + \frac{\partial}{\partial s}\right)\int_0^l\int_0^{2\pi} d\phi_o ds_o \rho_g(s_o)[L_\phi\cos(\phi-\phi_o) - L_t\sin(\phi-\phi_o)\sin\theta_g(s_o)]G\Bigg\}
\end{aligned}
\tag{154}
$$

As $\vec{r}$ approaches $S^{\pm}$ from outside S, the tangential components of $\vec{E}^{sc}(\vec{r})$ as given by eqs. (152) to (154) become:

$$
\begin{aligned}
E_\phi^{sc\pm}(\phi,s) &= \hat{\phi}\cdot\vec{E}^{sc\pm} \\
&= \frac{ik^2}{4\pi}\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[K_\phi \cos(\phi-\phi_o) - K_t \sin(\phi-\phi_o)\sin\theta_g(s_o)]G(\vec{r}-\vec{r}_o) \\
&- \frac{i}{4\pi\rho_g(s)}\frac{\partial}{\partial\phi}\int_0^l\int_0^{2\pi} ds_o d\phi_o [K_\phi \frac{\partial}{\partial\phi_o}G(\vec{r}-\vec{r}_o) + \rho_g(s_o)K_t\frac{\partial}{\partial s_o}G(\vec{r}-\vec{r}_o)] \\
&- \frac{k}{4\pi}\frac{\partial}{\partial s}\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[L_\phi \sin(\phi-\phi_o)\cos\theta_g(s) \\
&+ L_t\cos(\phi-\phi_o)\cos\theta_g(s)\sin\theta_g(s_o) - L_t\sin\theta_g(s)\cos\theta_g(s_o)]G(\vec{r}-\vec{r}_o) \\
&- \frac{k}{4\pi}\left[\kappa_g(s) - \left.\frac{\partial}{\partial n}\right|_{S^\pm}\right]\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[L_t\cos(\phi-\phi_o)\sin\theta_g(s)\sin\theta_g(s_o) \\
&+ L_t\cos\theta_g(s)\cos\theta_g(s_o) + L_\phi\sin(\phi-\phi_o)\sin\theta_g(s)]G(\vec{r}-\vec{r}_o)
\end{aligned}
\tag{155}
$$

$$
\begin{aligned}
E_t^{sc\pm}(\phi,s) &= \hat{t}\cdot\vec{E}^{sc\pm} \\
&= \frac{ik^2}{4\pi}\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[K_\phi \sin(\phi-\phi_o)\sin\theta_g(s) \\
&+ K_t\cos(\phi-\phi_o)\sin\theta_g(s)\sin\theta_g(s_o) + K_t\cos\theta_g(s)\cos\theta(s_o)]G(\vec{r}-\vec{r}_o) \\
&- \frac{i}{4\pi}\frac{\partial}{\partial s}\int_0^l\int_0^{2\pi} ds_o d\phi_o [K_\phi\frac{\partial}{\partial\phi_o}G(\vec{r}-\vec{r}_o) + \rho_g(s_o)K_t\frac{\partial}{\partial s_o}G(\vec{r}-\vec{r}_o)] \\
&+ \frac{k}{4\pi\rho_g(s)}\frac{\partial}{\partial\phi}\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[L_\phi\sin(\phi-\phi_o)\cos\theta_g(s) \\
&- L_t\sin\theta_g(s)\cos\theta_g(s_o) + L_t\cos(\phi-\phi_o)\cos\theta_g(s)\sin\theta_g(s_o)]G(\vec{r}-\vec{r}_o) \\
&- \frac{k}{4\pi}\left[\frac{\cos\theta_g(s)}{\rho_g(s)} + \left.\frac{\partial}{\partial n}\right|_{S^\pm}\right]\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)\,[L_\phi\cos(\phi-\phi_o) \\
&- L_t\sin(\phi-\phi_o)\sin\theta_g(s_o)]G(\vec{r}-\vec{r}_o)
\end{aligned}
\tag{156}
$$

where $\left.\frac{\partial}{\partial n}\right|_{S^\pm}$ is the normal derivative taken in the limit as $\vec{r}$ approaches the surface S^+ or S^- respectively from outside of S. It is evaluated as a limiting value in this report and should not be confused with the Fourier index n. From the duality principle, the tangential components

of $\vec{H}^{sc\pm}$ can be obtained by replacing $\vec{E}^{sc\pm}$ with $\vec{H}^{sc\pm}$, $\vec{K}$ with $\vec{L}$, and $\vec{L}$ with $-\vec{K}$:

$$\begin{aligned}
H_{\phi}^{sc\pm}(\phi,s) = {} & \frac{ik^2}{4\pi}\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[L_{\phi}\cos(\phi-\phi_o) - L_t \sin(\phi-\phi_o)\sin\theta_g(s_o)]G(\vec{r}-\vec{r}_o) \\
& - \frac{i}{4\pi\rho_g(s)}\frac{\partial}{\partial\phi}\int_0^l\int_0^{2\pi} ds_o d\phi_o [L_{\phi}\frac{\partial}{\partial\phi_o}G(\vec{r}-\vec{r}_o) + \rho_g(s_o)L_t\frac{\partial}{\partial s_o}G(\vec{r}-\vec{r}_o)] \\
& + \frac{k}{4\pi}\frac{\partial}{\partial s}\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[K_{\phi}(\phi_o,s_o)\ \sin(\phi-\phi_o)\cos\theta_g(s) \\
& + K_t\cos(\phi-\phi_o)\cos\theta_g(s)\sin\theta_g(s_o) - K_t(\phi_o,s_o)\sin\theta_g(s)\cos\theta_g(s_o)]G(\vec{r}-\vec{r}_o) \\
& + \frac{k}{4\pi}\left[\kappa_g(s) - \left.\frac{\partial}{\partial n}\right|_{S^\pm}\right]\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[K_t\cos(\phi-\phi_o)\sin\theta_g(s)\sin\theta_g(s_o) \\
& + K_t\cos\theta_g(s)\cos\theta_g(s_o) + K_{\phi}\sin(\phi-\phi_o)\sin\theta_g(s)]G(\vec{r}-\vec{r}_o)
\end{aligned} \tag{157}$$

$$\begin{aligned}
H_t^{sc\pm}(\phi,s) = {} & \frac{ik^2}{4\pi}\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[L_{\phi}\sin(\phi-\phi_o)\sin\theta_g(s) \\
& + L_t\cos(\phi-\phi_o)\sin\theta_g(s)\sin\theta_g(s_o) + L_t\cos\theta_g(s)\cos\theta(s_o)]G(\vec{r}-\vec{r}_o) \\
& - \frac{i}{4\pi}\frac{\partial}{\partial s}\int_0^l\int_0^{2\pi} ds_o d\phi_o [L_{\phi}\frac{\partial}{\partial\phi_o}G(\vec{r}-\vec{r}_o) + \rho_g(s_o)L_t\frac{\partial}{\partial s_o}G(\vec{r}-\vec{r}_o)] \\
& - \frac{k}{4\pi\rho_g(s)}\frac{\partial}{\partial\phi}\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)[K_{\phi}\sin(\phi-\phi_o)\cos\theta_g(s) \\
& - K_t\sin\theta_g(s)\cos\theta_g(s_o) + K_t\cos(\phi-\phi_o)\cos\theta_g(s)\sin\theta_g(s_o)]G(\vec{r}-\vec{r}_o) \\
& + \frac{k}{4\pi}\left[\frac{\cos\theta_g(s)}{\rho_g(s)} + \left.\frac{\partial}{\partial n}\right|_{S^\pm}\right]\int_0^l\int_0^{2\pi} ds_o d\phi_o \rho_g(s_o)\ [K_{\phi}\ \cos(\phi-\phi_o) \\
& - K_t\sin(\phi-\phi_o)\sin\theta_g(s_o)]G(\vec{r}-\vec{r}_o)
\end{aligned} \tag{158}$$

Because of the rotational symmetry of the scatterer, $G(\vec{r}-\vec{r}_o) = \sum_{n=-\infty}^{\infty} e^{in(\phi-\phi_o)}G_n(\rho,z;\rho_o,z_o)$.

The Fourier expansions of eqs. (156) and (157) are:

$$
\begin{aligned}
E^{sc\pm}_{\phi,n}(s) = {} & \frac{i}{2}\int_o^l ds_o\Big[\frac{k^2}{2}\,\rho_g(s_o)(G_{n+1}+G_{n-1}) - \frac{n^2}{\rho_g(s)}\,G_n\Big]\,K_{\phi,n} \\
& + \frac{1}{2}\int_o^l ds_o\rho_g(s_o)\Big[\frac{k^2}{2}\sin\theta_g(s_o)(G_{n+1}-G_{n-1}) + \frac{n}{\rho_g(s)}\frac{\partial}{\partial s_o}\,G_n\Big]\,K_{t,n} \\
& - \frac{ik}{4}\,\frac{\partial}{\partial s}\int_o^l ds_o\rho_g(s_o)\,\cos\theta_g(s)(G_{n+1}-G_{n-1})\,L_{\phi,n} \\
& - \frac{ik}{4}\left[\kappa_g(s) - \frac{\partial}{\partial n}\bigg|_{S^\pm}\right]\int_o^l ds_o\rho_g(s_o)\,\sin\theta_g(s)\,(G_{n+1}-G_{n-1})\,L_{\phi,n} \\
& + \frac{k}{2}\,\frac{\partial}{\partial s}\int_o^l ds_o\rho_g(s_o)\,[\sin\theta_g(s)\cos\theta_g(s_o)G_n \\
& \qquad - \frac{1}{2}\cos\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1})]L_{t,n} \\
& - \frac{k}{2}\left[\kappa_g(s) - \frac{\partial}{\partial n}\bigg|_{S^\pm}\right]\int_o^l ds_o\rho_g(s_o)[\cos\theta_g(s)\cos\theta_g(s_o)G_n \\
& \qquad + \frac{1}{2}\sin\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1})]L_{t,n}
\end{aligned}
\tag{159}
$$

$$
\begin{aligned}
E^{sc\pm}_{t,n}(s) = {} & -\frac{n}{2}\frac{\partial}{\partial s}\int_o^l ds_o G_n K_{\phi,n} - \frac{k^2}{4}\int_o^l ds_o\rho_g(s_o)\sin\theta_g(s)(G_{n+1}-G_{n-1})K_{\phi,n} \\
& - \frac{i}{2}\,\frac{\partial}{\partial s}\int_o^l ds_o\,\rho_g(s_o)\,\Big[\frac{\partial}{\partial s_o}\,G_n\Big]\,K_{t,n} \\
& + \frac{ik^2}{2}\int_o^l ds_o\,\rho_g(s_o)\,[\cos\theta_g(s)\cos\theta_g(s_o)\,G_n \\
& \qquad + \frac{1}{2}\sin\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1})]\,K_{t,n} \\
& - \frac{k}{4}\left[\frac{\cos\theta_g(s)}{\rho_g(s)} + \frac{\partial}{\partial n}\bigg|_{S^\pm}\right]\int_o^l ds_o\,\rho_g(s_o)\,(G_{n+1}+G_{n-1})\,L_{\phi,n} \\
& - \frac{kn\cos\theta_g(s)}{4\rho_g(s)}\int_o^l ds_o\,\rho_g(s_o)\,(G_{n+1}-G_{n-1})\,L_{\phi,n} \\
& + \frac{ik}{4}\left[\frac{\cos\theta_g(s)}{\rho_g(s)} + \frac{\partial}{\partial n}\bigg|_{S^\pm}\right]\int_o^l ds_o\,\rho_g(s_o)\,\sin\theta_g(s_o)(G_{n+1}-G_{n-1})\,L_{t,n} \\
& + \frac{ikn}{2\rho_g(s)}\int_o^l ds_o\,\rho_g(s_o)\,\Big[\frac{1}{2}\cos\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1}) \\
& \qquad - \sin\theta_g(s)\cos\theta_g(s_o)\,G_n\Big]\,L_{t,n}
\end{aligned}
\tag{160}
$$

Similarly, the Fourier expansions of eqs. (158) and (159) are:

$$
\begin{aligned}
H^{sc\pm}_{\phi,n}(s) \;=\;& \frac{i}{2}\int_o^l ds_o[\frac{k^2}{2}\,\rho_g(s_o)(G_{n+1}+G_{n-1}) \;-\; \frac{n^2}{\rho_g(s)}\;G_n]\;L_{\phi,n}\\
&+\;\frac{1}{2}\int_o^l ds_o\rho_g(s_o)[\frac{k^2}{2}\sin\theta_g(s_o)(G_{n+1}-G_{n-1})+\frac{n}{\rho_g(s)}\frac{\partial}{\partial s_o}\;G_n]\;L_{t,n}\\
&+\;\frac{ik}{4}\;\frac{\partial}{\partial s}\;\int_o^l ds_o\rho_g(s_o)\;\cos\theta_g(s)(G_{n+1}-G_{n-1})\;K_{\phi,n}\\
&+\;\frac{ik}{4}\left[\kappa_g(s)\;-\;\frac{\partial}{\partial n}\bigg|_{S^\pm}\right]\int_o^l ds_o\rho_g(s_o)\;\sin\theta_g(s)\;(G_{n+1}-G_{n-1})\;K_{\phi,n}\\
&-\frac{k}{2}\frac{\partial}{\partial s}\int_o^l ds_o\rho_g(s_o)\,[\sin\theta_g(s)\cos\theta_g(s_o)G_n\\
&\qquad\qquad -\frac{1}{2}\cos\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1})]K_{t,n}\\
&+\;\frac{k}{2}\left[\kappa_g(s)\;-\;\frac{\partial}{\partial n}\bigg|_{S^\pm}\right]\int_o^l ds_o\rho_g(s_o)\;[\cos\theta_g(s)\cos\theta_g(s_o)G_n\\
&\qquad\qquad +\frac{1}{2}\sin\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1})]\;K_{t,n}
\end{aligned}
\tag{161}
$$

$$
\begin{aligned}
H^{sc\pm}_{t,n}(s) \;=\;& -\frac{n}{2}\frac{\partial}{\partial s}\int_o^l ds_oG_nL_{\phi,n}-\frac{k^2}{4}\int_o^l ds_o\rho_g(s_o)\sin\theta_g(s)(G_{n+1}-G_{n-1})L_{\phi,n}\\
&-\;\frac{i}{2}\;\frac{\partial}{\partial s}\;\int_o^l ds_o\;\rho_g(s_o)\;[\frac{\partial}{\partial s_o}\;G_n]\;L_{t,n}\\
&+\;\frac{ik^2}{2}\;\int_o^l ds_o\;\rho_g(s_o)\;[\cos\theta_g(s)\cos\theta_g(s_o)\,G_n\\
&\qquad\qquad +\frac{1}{2}\sin\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1})]\;L_{t,n}\\
&+\;\frac{k}{4}\left[\frac{\cos\theta_g(s)}{\rho_g(s)}\;+\;\frac{\partial}{\partial n}\bigg|_{S^\pm}\right]\int_o^l ds_o\;\rho_g(s_o)\;(G_{n+1}\;+\;G_{n-1})\;K_{\phi,n}\\
&+\;\frac{kn\cos\theta_g(s)}{4\rho_g(s)}\;\int_o^l ds_o\;\rho_g(s_o)\;(G_{n+1}\;-\;G_{n-1})\;K_{\phi,n}\\
&-\;\frac{ik}{4}\left[\frac{\cos\theta_g(s)}{\rho_g(s)}\;+\;\frac{\partial}{\partial n}\bigg|_{S^\pm}\right]\int_o^l ds_o\;\rho_g(s_o)\;\sin\theta_g(s_o)(G_{n+1}\;-\;G_{n-1})\;K_{t,n}\\
&-\;\frac{ikn}{2\rho_g(s)}\;\int_o^l ds_o\;\rho_g(s_o)\;[\frac{1}{2}\;\cos\theta_g(s)\sin\theta_g(s_o)(G_{n+1}+G_{n-1})\\
&\qquad\qquad -\sin\theta_g(s)\cos\theta_g(s_o)\;G_n]\;K_{t,n}
\end{aligned}
\tag{162}
$$

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